Unsuspectable connections between Macrocosm and Microcosm
Part I

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Abstract

In a previous paper [6] we already showed the strict connection existing between certain macroscopic physical properties of a given compound (like the speed of sound and Young's modulus of elasticity) and some atomic features like the electron charge, the atomic mass, and the intermolecular distance. Here those relations are generalized and similar connections are found for the thermal expansion coefficients and the surface tension of liquids with the same atomic properties. This seems to imply that the various phases of matter are essentially due to ionic forces that appear even outside the bodies and are in agreement with a well-known experience by Ettore Majorana.

1 Introduction

In paper [6] we already highlighted that some macroscopic properties of matter can be associated to its atomic features and *vice versa*. Here we add further deductions and, to give an organic picture of our results, we report a very brief summary of the starting points already outlined in that paper.

A criticism that can be done to the elasticity theory, for example, and to the field concept, more generally, is the following. Usually, given a certain substance, we admit that any amount $(\Delta x)^3$ of this substance can be considered at a scale smaller and smaller without observing changes in its structure, i.e. without reaching and observing finally its crystal atomic lattice. This hypothesis implies that a set of important macroscopic formulas (e.g. the well-known relation that makes it possible to evaluate the speed of sound within the matter) will remain unchanged at any scale without transforming into relations concerning interatomic and molecular structures and bonds, that at a certain scale should substitute the continuum (^{1}). Just to fix ideas, to obtain the well-known relation

$$\frac{\partial^2 z}{\partial s^2} = C^2 \frac{\partial^2 z}{\partial t^2} \tag{1}$$

concerning the propagation of an elastic deformation in a solid or a gas, let us consider an infinitesimal volume of matter with a given density, and let us apply the second law of dynamics. Nothing prevents us to consider, for a concrete example, the case of sodium chloride ($NaCl$), using a schematic representation where it is formed by a sequence of masses Na^+ and Cl^- connected by an elementary electrical charge (see Fig. 1).

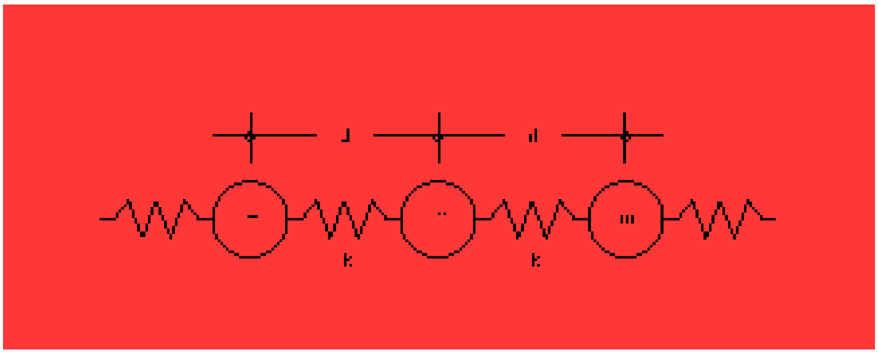

Fig 1

In such a case, we see that for a generic mass (^{2}) the following relation holds [2]:

$$m \frac{d^2 s_n}{dt^2} = -k(2s_n - s_{n+1} - s_{n-1}),$$

where s is the longitudinal movemente of the generic mass m and k is the elastic constant that unites two consecutive masses.

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[1] Another criticism arises when we observe that, in the analysis of the wave-corpuscle interaction, the matter has been always considered as a simple harmonic oscillator (electromagnetic dipole), neglecting the fundamental non-linearity of Coulomb's law. This point is analyzed in detail in other papers [9].

[2] The chlorine mass is not equal to the sodium mass. This point is solved by introducing the reduced mass μ.

The solution [2] shows that an elastic wave propagates along the sequence of masses, and that the frequency of this wave is

$$\nu = \frac{1}{\pi}\sqrt{\frac{k}{m}}\sin\frac{\pi d}{\lambda}. \tag{2}$$

When the wavelength is greater than the interatomic distance, we have the simpler relation:

$$\nu \simeq \frac{d}{\lambda}\sqrt{\frac{k}{m}} \tag{3}$$

and thus we deduce that

$$C = \nu\lambda = d\sqrt{\frac{k}{m}}. \tag{4}$$

If $d \times d = S$ is the normal section of the sequence of masses, each of them will be contained in a volume Sd, the density will be given by

$$\rho = \frac{m}{Sd},$$

and (4) becomes

$$C = \sqrt{\frac{dk}{\rho S}}.$$

Due to Hooke's law, if we use the symbol E_Y to denote the modulus of elasticity of the material, the relative shift (s/d) will be given by

$$\frac{s}{d} = \frac{F}{E_Y S} = \frac{ks}{E_Y S} \tag{5}$$

and thus, finally, (4) transforms into

$$C = \sqrt{\frac{E_Y}{\rho}}. \tag{6}$$

In the specific case of the sodium chloride crystal [2], if the two ion systems shift by a distance d towards opposite directions, then the force between them will be given by

$$F = 2kd.$$

Taking into account the electric bond existing between the two ions, we find that (³)

$$k = \frac{e^2}{d^3}, \tag{7}$$

or

$$F = \frac{2e^2}{d^2}. \tag{8}$$

Moreover, if the force given by relation (8) acts on the lattice surface $d \times d$ (⁴), Young's modulus of elasticity E_Y will be equal to

$$E_Y = \frac{2e^2}{d^4}. \tag{9}$$

This implies that the speed of sound C in the considered material is given by

$$C = \nu\lambda = d\sqrt{\frac{2k}{m}} = d\sqrt{\frac{2e^2}{md^3}} = \sqrt{\frac{2e^2}{md}}. \tag{10}$$

Thus, this speed is equal to a certain *orbital* speed (velocità di fuga) of two consecutive masses electrically bound each other.

On the other hand, the last relation can be written in the form

$$C = \sqrt{\frac{2e^2}{md}} = \sqrt{\frac{2e^2}{\frac{m}{d^3}d^4}} = \sqrt{\frac{2e^2}{\rho d^4}} = \sqrt{\frac{E_Y}{\rho}}, \tag{11}$$

from which we see the procedure that can be used to recover the formula found with the continuum hypothesis, starting from the atomic discontinuity. All of this can be generalized to the case in which the consecutive atomic masses are not equal. In such a case, introducing the reduced mass, we have

$$C = \sqrt{\frac{2e^2}{\mu d}} \tag{12}$$

and

$$E_Y = \frac{2e^2}{d^4}. \tag{13}$$

Even the crystal pulsation can be deduced easily, and is given by

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[3] This is a quite simple way to introduce atomic quantities.

[4] Actually the generic atom is located in a cubic cell whose side is d.

$$\omega = \frac{C}{d} = \sqrt{\frac{2e^2}{\mu\ d^3}} \tag{14}$$

2 Experimental checks

All the formulas previously deduced can be tested experimentally if we take into account that:

- they were deduced in the context of a model that can be improved further (as we will see later);
- they are valid only for 100% pure materials, i.e. materials without defects and dislocations;
- the data used for the interatomic distances, the modulus of elasticity, and the speed of sound are not completely consistent each other, because they were extracted from tables compiled by means of various experiments, performed with distinct aims.

1) The example that we discuss now concerns the absorption of light by matter. The matter can be idealized like a set of harmonic oscillators (an extended and more complete model should consider even the anharmonic or non-linear oscillators [11, 9] that resonate when they are reached by radiation).

According to classical electromagnetism, the radiation emitted or absorbed must have the same pulsation of the oscillating charge. Thus, it is possible to evaluate the pulsation of the substance under study by measuring the pulsation of the emitted radiation. We report now detailed calculations taken from [2] (in italic fonts). It is easy to see that they use the experimental value of the modulus of elasticity, while we will make recourse to the theoretical formula (14).

Let m_1 *and* m_2 *be the masses of two ions and let* k_o *be the elastic constant of the force existing between two consecutive ions. When the* Na^+ *and* Cl^- *ion systems moves towards opposite directions with a shift* x*, each ion undergoes a force* $2k_o x$*, and therefore the pulsation is given by the following relation:*

$$\omega_o = \sqrt{\frac{2k_o}{\mu}} \tag{15}$$

where $\mu = m_1 m_2/(m_1 + m_2)$ *is the reduced mass. The coefficient* k_o *can be expressed as a function of the elastic coefficients of the crystal: in fact, if* d

is the distance between two consecutive ions (it can be evaluated through an X-ray analysis of the sample), there are $1/d$ *ions per unit length and* $1/d^2$ *ions per unit surface (for surfaces orthogonal to the shift).*

Thus we find

$$k_o = \frac{d^2}{d} c_{11} = dc_{11}$$

where c_{11} *is the elastic coefficient that relates the force to the shift* $x'x$ *that occurs in the same direction. We see immediately that relation (15) is in agreement with the data reported in the following table.*

Crystal	c_{11} (N/m^2)	d (m)	ω_o (sec^{-1}) (calculated)	ω_o (sec^{-1}) (measured)
$NaCl$	4.9×10^{10}	2.8×10^{-10}	3.3×10^{13}	3.2×10^{13}
KCl	4.0×10^{10}	3.1×10^{-10}	2.4×10^{13}	2.7×10^{13}
KBr	3.5×10^{10}	3.3×10^{-10}	2.3×10^{13}	2.1×10^{13}

Let us observe that the coefficients c_{11} are evaluated experimentally by means of laboratory tests. On the other hand, according to the previous discussions, we can assume that the two ions are electrically bound, and can deduce the previous pulsations without making recourse to the experimental measurement of Young's modulus of elasticity.

Setting

$e = 4.8 \times 10^{-10}$ u.e.;
mass of $Na = 23 \times (1.673 \times 10^{-24})$ g;
mass of $Cl = 35.5 \times (1.673 \times 10^{-24})$ g;
$d = 2.8 \times 10^{-8}$ cm;

for $NaCl$ we get

$\mu = \frac{23 \times 35.5}{(23+35.5)} 1.673 \times 10^{-24} = 2.3 \times 10^{-23}$ g.

Applying relation (14) we obtain (^{5})

$$\omega_{o\ NaCl} = \sqrt{\frac{2\ e^2}{\mu d^3}} = \sqrt{\frac{2\ (4.8 \times 10^{-10})^2}{2.3 \times 10^{-23}\ (2.8 \times 10^{-8})^3}} = 3 \times 10^{13}\ \text{sec}^{-1}. \qquad (16)$$

Analogously, for KCl:

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[5] Interatomic distances are determined by means of X-ray experiments.

mass of $K = 39.1 \times (1.673 \times 10^{-24})$ g;
mass of $Cl = 35.5 \times (1.673 \times 10^{-24})$ g;
$d = 3.1 \times 10^{-8}$ cm;
$\mu = \frac{39.1\times\ 35.5}{(39.1+35.5)} 1.673 \times 10^{-24} = 3.11 \times 10^{-23}$ g.

we have

$$\omega_{o\ KCl} = \sqrt{\frac{2\ e^2}{mp^3}} = \sqrt{\frac{2\ e^2}{\mu d^3}} = \sqrt{\frac{2\ (4.8\times 10^{-10})^2}{3.11\times 10^{-23}\ (3.1\times 10^{-8})^3}} = 2.2\times 10^{13}\ \mathrm{sec}^{-1}. \tag{17}$$

And for KBr:

mass of $K = 39.1 \times (1.673 \times 10^{-24})$ g;
mass of $Br = 80 \times (1.673 \times 10^{-24})$ g;
$d = 3.3 \times 10^{-8}$ cm;
$\mu = \frac{39.1\ 80}{(39.1+80)} 1.673 \times 10^{-24} = 4.4 \times 10^{-23}$ g;

we have:

$$\omega_{o\ KBr} = \sqrt{\frac{2\ e^2}{mp^3}} = \sqrt{\frac{2\ e^2}{\mu d^3}} = \sqrt{\frac{2\ (4.8\times 10^{-10})^2}{4.4\times 10^{-23}\ (3.3\times 10^{-8})^3}} = 1.7\times 10^{13}\ \mathrm{sec}^{-1}. \tag{18}$$

If we compare these theoretical values with the experimental values, we note that they have the same order of magnitude. It is important to observe that, using the experimental values of c_{11} we obtain a better agreement, but this is due to the fact that the theoretical model is very simple and thus does not take into account the presence of defects in the material, the anharmonicity, the viscosity, and so on. All of these are effects that have a not-negligible influence. On the other hand, we will see that the model used can be improved.

2) Now, let us determine the speed of sound in an aluminum rod. We can write (formula (12) :

$$C = \sqrt{\frac{2\times (4.8\times 10^{-10})^2}{26.98\times 1.673\times 10^{-24}\times (2.86\times 10^{-8})}} = 5.9\times 10^5\ \mathrm{cm/sec}.$$

This gives a value comparable with the experimental value 5.1×10^5 cm/sec.

For lead we have:

$$C = \sqrt{\frac{2\times (4.8\times 10^{-10})^2}{207.21\times 1.673\times 10^{-24}\times (3.49\times 10^{-8})}} = 1.9\times 10^5\ \mathrm{cm/sec},$$

to be compared with the experimental value 1.2 × 10^5 cm/sec. We will see later a way to improve these formulas.

3) We can hypothesize the existence of an electric bond even between the molecules of a gas. Let us consider the oxygen at 273°K. In such a case we have ([6]):

$$\begin{aligned} C_{O_2} = \sqrt{\frac{2e^2}{md}} = \sqrt{\frac{2 \times (4.8 \times 10^{-10})^2}{5.34 \times 10^{-23} \times 6.2 \times 10^{-6}}} &= 37307 \text{ cm/sec} \\ &= 373 \text{ m/sec}, \end{aligned}$$

or, for the hydrogen at the same temperature:

$$\begin{aligned} C_{H_2} = \sqrt{\frac{2e^2}{md}} = \sqrt{\frac{2 \times (4.8 \times 10^{-10})^2}{3.37 \times 10^{-24} \times 8 \times 10^{-6}}} &= 1.307 \times 10^5 \text{ cm/sec} \\ &= 1307 \text{ m/sec}. \end{aligned}$$

These improvable values are of the same order of magnitude of the experimental values, that for oxygen and hydrogen are, respectively, equal to 317 m/sec and 1270 m/sec.([7]).

4) Modulus of elasticity of the homogeneous iron, aluminum, and copper. For the iron we have:

$$E_f = 2\frac{e^2}{d^4} = \frac{2\ (4.8 \times 10^{-10})^2}{(2.48 \times 10^{-8})^4} = 1.2 \times 10^{12} \text{ dyne/cm}^2 = 1.2 \times 10^6 \text{ Kg/cm}^2,$$

a value that is equal to the experimental value. In such a case we can observe that the steel, homogeneous iron with a small percentage of carbon added, has a corresponding value of 2.1 × 10^6 Kg/cm^2. It is possible to think that the addition of carbon modifies notably the interatomic distance of the homogeneous iron. It is easy to find that the interatomic distance should be equal to

$$d = \sqrt[4]{\frac{2 \times (4.8 \times 10^{-10})^2}{2.1 \times 10^{12}}} = 2.16 \times 10^{-8} \text{ cm},$$

with a variation of 0.3 × 10^{-8} cm.

For the aluminum we have:

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[6] For the air at standard pressure, the molecules have a mean free path (intermolecular distance) approximately equal to 6.2 × 10^{-6} cm [1, p. 32].

[7] As we're going to see of a more appropriate model improves the predictions.

$$E_a = 2\frac{e^2}{d^4} = \frac{2\ (4.8\times 10^{-10})^2}{(2.86\times 10^{-8})^4} = 6.8\times 10^{11}\ \text{dyne/cm}^2\ = 6.8\times 10^{5}\ \text{Kg/cm}^2,$$

that must be compared to the experimental value 6.5 10^5 Kg/cm^2.

For the copper we have:

$$E_r = 2\frac{e^2}{d^4} = \frac{2\ (4.8\times 10^{-10})^2}{(2.55\times 10^{-8})^4} = 1.09\times 10^{12}\ \text{dyne/cm}^2\ = 1.1\times 10^{6}\ \text{Kg/cm}^2,$$

while the experimental value is 1.17×10^6 Kg/cm^2.

The relations used will be improved in the following.

3 Thermal expansion

Let us consider, in a given crystal, a pair of consecutive atoms of the elementary cell, and assume that they are ionized only one time. Using the symbol e to denote the electron charge, and r to denote the mean intermolecular distance, we can say that the bonding energy of our pair of ions is given by

$$E = -\frac{1}{2}\frac{e^2}{r}. \tag{19}$$

From (19) we also have:

$$\frac{dE}{dr} \simeq \frac{\Delta E}{\Delta r} = \frac{1}{2}\frac{e^2}{r^2}, \tag{20}$$

from which we obtain the energy variation, that can be expressed in the form:

$$\Delta E = \frac{1}{2}\frac{e^2}{r}\frac{\Delta r}{r} = \frac{1}{2}\frac{e^2}{r}\varepsilon,$$

here we have set

$$\varepsilon = \frac{\Delta r}{r}$$

for the unit extension. Therefore we have:

$$\Delta E = \frac{1}{2}\frac{e^2}{r}\varepsilon. \tag{21}$$

If we imagine that this energy variation is due to a variation ΔT of the temperature of the environment surrounding the system, then (8) we should have

$$\Delta E = \frac{1}{2}\frac{e^2}{r}\varepsilon = \frac{1}{2}\ k\ \Delta T \tag{22}$$

where k is Boltzmann's constant. This implies that the thermal expansion coefficient of the crystal is given by

$$\alpha = \frac{\varepsilon}{\Delta T} = \frac{kr}{e^2}. \tag{23}$$

This formula can be experimentally verified and seems to be valid for solids, liquids, and gases, i.e. various phases of the matter that are in this way characterized by the common distance r. On the other hand, if we admit that a certain substance, in the solid state, has a crystal structure whose atoms are electrically bonded, it is not clear why in the liquid or gas state these bonds should disappear, being known that the range of the electrical field is unlimited.

In the [*c.g.s.*] unit system, $k = 1.36 \times 10^{-16}$ erg $\times(°\text{K})^{-1}$ and thus (23) becomes

$$\alpha = \frac{k}{e^2}r = 590.2\bar{7} \times r,$$

from which it follows that

Crystal	k/e^2 (cm^{-1})	r (cm)	α (calculated)	α (experimental)
Aluminum	590.277	2.86×10^{-8}	1.7×10^{-5}	2.0×10^{-5}
Copper	590.277	2.55×10^{-8}	1.5×10^{-5}	1.7×10^{-5}
Iron	590.277	2.48×10^{-8}	1.4×10^{-5}	1.2×10^{-5}

Let us observe that in an ideal gas at standard pressure the mean intermolecular distance at 273.2°K is 6.2×10^{-6} cm (air)[1, pag. 459]; in such a case we have

$$\alpha = \frac{k}{e^2}r = 590.2\bar{7} \times 6.2 \times 10^{-6} \simeq 3.66 \times 10^{-3} \simeq \frac{1}{273.2}, \tag{24}$$

which coincides in practice with the known value.

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[8] For the principle of energy equipartition [2, p. 279].

4 The equation of perfect gases

By definition an ideal gas is a gas such that its molecules do not interact each other. We will show the important role played by the electric field even in this case.

We know that a gas at 0°C has a thermal energy

$$E = \frac{1}{2}\, k\ 273.14 = \frac{1}{2}\ 1.36 \times 10^{-16} \times 273.14 = 1.85 \times 10^{-14}\ \text{erg}. \tag{25}$$

If it is true that this energy is in equilibrium with a ionic energy, the latter must have the value given by (25). For the air we have:

$$E = \frac{1}{2}\frac{e^2}{r} = \frac{1}{2}\frac{\left(4.8 \times 10^{-10}\right)^2}{6.2 \times 10^{-6}} = 1.\,85 \times 10^{-14}\ \text{erg}.$$

If, given a gas, we admit that it is formed by ions, just like in a crystal, and make the hypothesis that all its Coulombian energy is in equilibrium with the thermal energy, we can write:

$$\frac{1}{2} N\ \frac{e^2}{r} = \frac{1}{2} N\ k\ T,$$

where N is Avogadro's number. This can also be written in the form:

$$N\ \frac{e^2}{r}\ \frac{r^3}{r^3} = N\ \frac{e^2}{r^4}\ r^3\ = p\ V\ = N\ k\ T,$$

or else

$$p\ V = R\ T, \tag{26}$$

where R is the perfect gas constant. Boltzmann's constant can be evaluated from the equation:

$$\frac{e^2}{r} = kT,$$

taking into account that, at 273.14°K, the average distance between the ions is 6.2×10^{-6} cm. Indeed we have:

$$\frac{\left(4.8 \times 10^{-10}\right)^2}{6.2 \times 10^{-6} \times 273.14} = 1.36 \times 10^{-16}.$$

Let us briefly mention a possible change that can be applied to the perfect gas equation. It is evident that we can write

$$\begin{aligned}\Delta E &= -\frac{1}{2}e^2\left(\frac{1}{r_1}-\frac{1}{r_2}\right) = -\frac{1}{2}\frac{e^2}{r_1}\left(1-\frac{r_1}{r_2}\right) \\ &= -\frac{1}{2}\frac{e^2}{r_1}\frac{r_1^3}{r_1^3}\left(1-\frac{r_1}{r_2}\right) = -\frac{1}{2}pV\left(1-\frac{r_1}{r_2}\right),\end{aligned}$$

and therefore

$$\frac{1}{2}pV\left(1-\frac{r_1}{r_2}\right) = \frac{1}{2}Nk\Delta T,$$

or

$$pV\left(1-\frac{r_1}{r_2}\right) = R\ \Delta T. \tag{27}$$

Furthermore

$$\Delta E = \frac{1}{2}\frac{e^2}{r}\varepsilon = \frac{1}{2}pV\ \varepsilon = \frac{1}{2}k\Delta T,$$

or

$$pV\ \varepsilon = pV\ \alpha\ \Delta T = N\ k\ \Delta T,$$

from which, at standard temperature and pressure, we have:

$$\alpha = \frac{N\ k}{pV} = \frac{60.6\times 10^{22}\times 1.38\times 10^{-16}}{1.033\times 10^6\times 22414} = 3.589\,2\times 10^{-3} \simeq \frac{1}{276}. \tag{28}$$

By comparison between (24) and (28) we also deduce that

$$N = \frac{\frac{1}{2}pV}{\frac{1}{2}\frac{e^2}{r}}, \tag{29}$$

that is, the ratio between the macroscopic energy (pV) contained in a mole of gas and the microscopic electric energy $(\frac{1}{2}\frac{e^2}{r})$ of a single molecule is equal to Avogadro's number.

Ed infatti nelle ordinarie condizioni di pressione e temperatura si ha

$$N = \frac{1.033\times 10^6\times 22414}{\left(\frac{(4.8\times 10^{-10})^2}{6.2\times 10^{-6}}\right)} = 6.23\times 10^{23}. \tag{30}$$

5 Remark

We know that a charge vibrating with a certain frequency ν emits an electromagnetic radiation of the same frequency. This perfect identity is valid both in classic electromagnetism and in quantistic physics. Nothing is said about a possible bond between the vibration ampleness of the charge ψ and the electromagnetic wave length λ emitted by the same one. In the work [12] we establish that this relation is given by the formula

$$\lambda = 2\pi 137 \psi n,$$

where n is a determined quantistic number. In the said work we show, only in a theoretically way and in various ways, the validity of the previous relation and its compatibility with the actual quantistic mechanics. The things we're going to say now could constitute instead an easy experimental verification of these relation. In the cases in which we don't recur to accurate spectrographic analyses 31 can also be written

$$\lambda = 2\pi 137 \Psi = 2\pi 137 r$$

And we also have the identity [12]

$$\frac{e^2}{r} = \frac{e^2}{\Psi} = \frac{hC}{\lambda}.$$

Let's consider a gas now. The heat emitted by this gas is constituted by electromagnetic waves with a certain predominant wave lenght. We can write

$$\frac{e^2}{r} = \frac{e^2}{\Psi} = \frac{hC}{\lambda} = h\nu = kT.$$

If we hypothesize that the gas would be at a temperature of 273 $^\circ Kelvin$ we will have

$$\nu = \frac{kT}{h} = \frac{1.36 \times 10^{-16} \times 273}{6.67 \times 10^{-27}} = 5.56 \times 10^{12}$$

To whom corresponds an electromagnetic wave length equal to

$$\lambda = \frac{2.998 \times 10^{10}}{5.56 \times 10^{12}} = cm.0.0053.$$

If we divide this number for $2\pi 137$ we'll have

$$r = \Psi = \frac{0.0053}{2\pi 137} = cm.6.2 \times 10^{-6}$$

distance that coincides with the medium path of the gas molecule at that temperature. In the case of the gasse, the molecules are subjected to that we call thermal agitation. We are in presence of charges continually accelerated and decelerate and this, for the relation of Larmor, produces electromagnetic radiations that we commonly call heat.

6 Specific heats

Let us consider again the relation

$$\Delta E = \frac{1}{2}\frac{e^2}{r}\varepsilon.$$

Introducing the thermal expansion coefficient α, we also have:

$$\Delta E = \frac{1}{2}\frac{e^2}{r}\ \alpha\ \Delta T.$$

Denoting with the symbol m the mass of the atom, we find

$$c_V = \frac{1}{m}\frac{\Delta E}{\Delta T} = \frac{1}{2}\frac{e^2}{mr}\alpha.$$

If we consider that a (finite size) atom has 6 degrees of freedom, we deduce that

$$\bar{c}_V = 3\frac{e^2}{mr}\alpha. \tag{31}$$

Since

$$\alpha = \frac{kr}{e^2}, \tag{32}$$

at last we find

$$\bar{c}_V = 3\frac{k}{m}, \tag{33}$$

and therefore

$$m\ \bar{c}_V = 3k, \tag{34}$$

or ([9])

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[9] Observe that
$3 \times N \times k = 3 \times 60.6 \times 10^{23} \times 1.38 \times 10^{-16} =$
2.5088×10^{8} erg°K = 5.9923 cal°K.

$$N\ m\ \bar{c}_V = C_V = 3\ N\ k = 3\ R.$$

This relation is equal to the law of Dulong and Petit ([10]).

From (31), taking into account that the speed of sound in the matter is

$$v = \sqrt{\frac{2e^2}{mr}},$$

we also have

$$\bar{c}_V = 3\frac{e^2}{mr}\alpha = \frac{3}{2}\frac{2e^2}{mr}\alpha = \frac{3}{2}v^2\alpha, \tag{35}$$

from which it follows that

$$v = \sqrt{\frac{2}{3}\frac{\bar{c}_V}{\alpha}}. \tag{36}$$

For the air we have (formula (33))

$$\bar{c}_V = \frac{3 \times 1.38 \times 10^{-16}}{2 \times 16 \times 1.672 \times 10^{-24}} = 7.73 \times 10^6 \text{ erg}^\circ\text{K}$$

and therefore

$$v = \sqrt{\frac{2}{3}\frac{7.73 \times 10^6}{\frac{1}{273.2}}} = 37522 \text{ cm/s} = 375 \text{ m/s}.$$

This value is the same as the previous one, and in any case is higher than the real value. However we must observe that we started from a model formed by a sequence of masses interacting through electric forces. A more realistic model of crystal is formed by a spatial lattice for which the real potential, for an ion inside the crystal, is represented by an expression of the type [2, p. 265] [3] [4]

$$E = -\left(6 - \frac{12}{\sqrt{2}} + \frac{8}{\sqrt{3}} \pm ...\right)\frac{e^2}{r}\left(1 - \frac{1}{n}\right). \tag{37}$$

For face centered cubic crystals ($NaCl$) we have

$$E = -1.74755\frac{e^2}{r}\left(1 - \frac{1}{9}\right) = -1.553\ \frac{e^2}{r}.$$

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[10] Observe that the mass of a proton (or of a hydrogen atom) is given by $m = \frac{1}{N} = \frac{1}{6.02 \times 10^{23}} = 1.66 \times 10^{-24}$ g.
In fact 1 gram of hydrogen contains N atoms.

Therefore, a more reliable formula for the speed of sound in the air could be of this type:

$$v = \sqrt{\frac{1.553 \times e^2}{m\ r}}, \tag{38}$$

and therefore at 0°K we find

$$v = \sqrt{\frac{1.553 \times (4.8 \times 10^{-10})^2}{2 \times 16 \times 1.672 \times 10^{-24} \times 6.2 \times 10^{-6}}} = 32843 \text{ cm/s} = 328 \text{ m/s}$$

that should be compared with the experimental value (332 m/s).

Observe that (38) can be compared with the well-known classical formula

$$v = \sqrt{\frac{p}{\rho}\gamma}, \tag{39}$$

where p represents the pressure in the generic gas, ρ represents the density, and $\gamma = C_p/C_V$ depends on the type of gas (for a monoatomic gas $\gamma = \frac{5}{3}$, for a biatomic gas $\gamma = \frac{7}{5}$, etc.). In fact (38) can be written in the form

$$v = \sqrt{\frac{e^2}{mr}\gamma'} = \sqrt{\frac{e^2}{\frac{m}{r^3}r^4}\gamma'} = \sqrt{\frac{p}{\rho}\gamma'}. \tag{40}$$

where, this time, γ' is the Madelung constant [4] that also depends (in a more precise form) on the number and the spatial pattern formed by the ions that constitute the substance under study. The previous formula can also be written in the form

$$v = \sqrt{\frac{e^2}{mr}\gamma'} = \sqrt{\frac{kT}{m}\gamma'} = \sqrt{\frac{RT}{Nm}\gamma'}, \tag{41}$$

that coincides with Laplace expression if $\gamma = \gamma'$. Let us observe that (41) also implies:

$$\frac{mv^2}{e^2/r} = \gamma'. \tag{42}$$

7 Surface tension and vaporization heat

In Fig. 2 we show the well-known table which makes it possible to deduce the value of the *surface* tension of a mixture of soap and water.

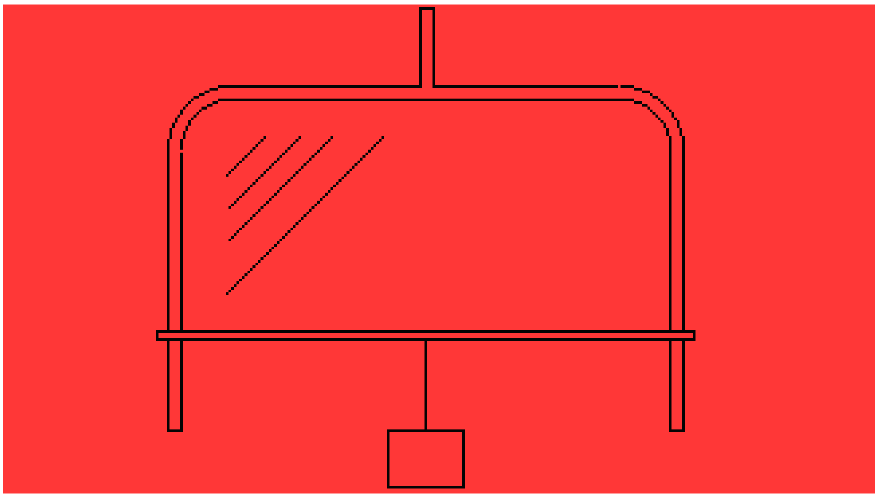

Fig. 2

We have already saw that the average intermolecular distance has an order of magnitude of 10^{-8} cm in solid substances, 10^{-7} cm in liquid substances, and 10^{-6} cm in a gas. If we take into account that even in the water the molecules have an average distance between them, it follows that the linear tension τ should be evidently given by a formula like

$$\tau = \frac{e^2}{d^3}\frac{s}{d}, \tag{43}$$

where d is the intermolecular distance and s is the thickness of the film of water. We do not know the distance d for the water in normal conditions; however, in a first instance, we can assume a value $d = 3 \times 10^{-7}$ cm and, for the average thickness, a value $s = 2.54 \times 10^{-6}$ cm ([11]). So we have

$$\tau = \frac{(4.8 \times 10^{-10})^2}{(3 \times 10^{-7})^4} 2.54 \times 10^{-6} = 72 \text{ dyn/cm}, \tag{44}$$

that practically agrees with the experimental value of approximately 76 dyn/cm.

We can evaluate the value of d by means of the following deduction. To vaporize completely 1 Kg of water, it is necessary to use, for each molecule, an energy equal to ([12])

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[11] The thickness of the film can be determined easily with the well-known light interference methods and the literature reports for it an average value of the order of 10^{-6} inches.

[12] We consider a non-polarized molecule.

$$\Delta E = e^2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = \frac{e^2}{r_1} \left(1 - \frac{r_1}{r_2} \right) , \tag{45}$$

where r_1 represents the average intermolecular distance for liquid water and r_2 the average intermolecular distance of vapor. If r_2 is of the order of 6×10^{-6} cm or higher, we see that (45) assumes the simpler form

$$\Delta E \simeq \frac{e^2}{r_1}. \tag{46}$$

Since 1 cm^3 of matter in normal conditions contains about 2.705×10^{19} molecules [8] and 1 Kg of water occupies approximately 10^3 cm^3, we have:

$$\Delta E \simeq 2.705 \times 10^{19} \times \frac{(4.8 \times 10^{-10})^2}{3 \times 10^{-7}} \times 1000 = 2.0774 \times 10^{10} \text{ erg} = 537 \text{ cal.}$$

This value agrees with the experimental value of 540 cal [8, p. 663]. Thus the previous evaluation is substantially confirmed until a more precise experimental verification. Another similar example concerns the evaluation of the energy needed for the creation or, equivalently, the destruction of a crystal structure. The most precise formula can be obtained for 1 Kmole of $NaCl$ [3, p. 266]:

$$\Delta E = 1.744 \frac{(4.8 \times 10^{-10})^2}{2.8 \times 10^{-8}} (1 - \frac{1}{9}) = 7.730\,2 \times 10^{12} \text{ erg} = 184 \text{ Kcal,}$$

that must be compared with the experimental value of 181 Kcal.

8 Weak ionic forces

Let us concentrate, now, on a concrete reasoning. We will consider a rod of homogeneous iron whose section is 1 cm^2. If we apply two forces having equal intensity and contrary directions to its ends, within the limits of elastic deformation, we will have ([13])

$$\sigma = E_Y \, \varepsilon \tag{47}$$

where Young's modulus of elasticity E_Y is given by the relation

$$E_Y \simeq 2 \frac{e^2}{d^4}. \tag{48}$$

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[13] Also this formula can be obtained easily from (21).

It is quite clear that, if we increase the tension applied, we have an increase (in the same direction) of the distance d, so that the force to be applied decreases correspondingly. There is no reason to think that the tension, well detectable when the sample is still intact, exists no more when the sample breaks in two pieces and these are positioned at any distance between them. The tension will be very weak, but it should still exist because the range of the electric field is unbounded. If, after the sample has been broken, we force the two pieces one against the other, we will see that, independently from the presence of the small fracture, a certain (weak) force is always needed to separate again the pieces. This is an effect of the cohesion force due to the presence of surface ions. The existence of these, still active, ions is much more evident in the well-known phenomena involving the surface tension. When two drops of water nearly touch they tend to join together abruptly and form a single drop. The adsorption phenomena of gases into metals, with the creation, for example, of molecules of iron oxide (new chemical bonds), are due to surface tension effects that can be associated completely to the existence of residual Coulombian fields on the surfaces of the bodies involved. We cannot exclude that ions contained in the body could still exert a residual action outside the body.

Thus, if a certain continuity principle is valid, the tension forces between solids, liquids, solids and liquids, and so on, do not appear only when they come in contact each other ([14]), but even when they are separated by common macroscopic distances.

Since experimental data concerning the transmission modes of this kind of forces at large distances are still lacking, we think that it is not the case to make hazardous estimates.

The first evidence concerning the experimental existence of this kind of forces, that should appear between bodies at large distance even if they are at the same potential, can be associated to an experiment by Ettore Majorana (see Fig. 3).

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[14] Here it would be necessary to define in a more precise way the meaning of the expression "contact distance".

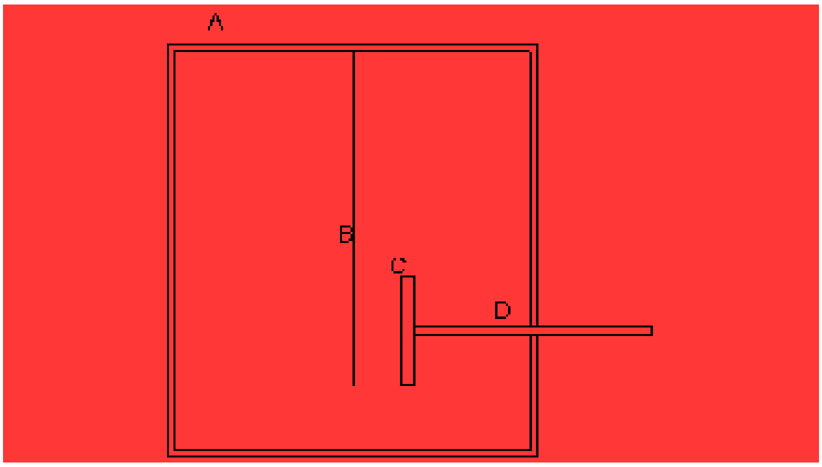

Fig. 3

E. Perucca [6, p. 687] reports the following description of the experiment and its results:

A thread B *of silica glass recovered by a metal, for example gold, hangs down vertically and is parallel to a metal plate* C, *and is electrically connected to a cage* A. *The distance between* C *and* B *can be set by means of a micrometrical screw; also* C *is electrically connected to the cage* A. *The experience shows that, if* C *is a metal different from the metal that recovers the silica thread* B, *at a specific distance between* C *and* B *(approximately 0.1 cm), the thread is notably attracted by* C *and moves speedy touching it. The adhesion (or cohesion that would appear even if* C *and* B *were made of the same metal) actually appears only at distances much smaller than this. The attraction observed is of electrical nature and shows the existence of facing charges with distinct signs on* C *and* B, *and thus the existence of a potential difference in the dielectric between the metals* C *and* B, *even if these metals are at the same potential.*

It would be of great interest to ascertain experimentally this potential difference (with a modified Majorana instrument), to compare it to the theoretical potential difference that can be evaluated by the knowledge of interatomic distances in the crystal structure of the two metals. In such a case we would also take into account that, at standard temperature, a potential difference (Melloni's thermocouple) appears between two metals with different crystal structure, whose value should be approximately

$$\Delta V = \frac{e}{d_1}\left(1 - \frac{d_1}{d_2}\right).$$

9 Note

In this paper we have only formulated the starting idea for further work, that of course can be made much more perfect. On the other hand, the various checks reported in the text have, at the present moment, an almost presumptive value because, for example, the modulus of elasticity of a substance mentioned in a book certainly was not measured on the same sample used for the evaluation of the interatomic distance. This implies that a number of more precise and aimed experiments are needed, together with a deeper theoretical research. Readers who would like to study these subjects further can e-mail to the Author at the following address: **carlo.santagata@tin.it**.

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