CRITICS TO THE PROBLEM OF THE TWO BODIES

THE ADVANCEMENT OF MERCURY'S PERIHELION

SIDEREAL TIME

LUNISOLAR PRECESSION AND PLATONIC YEARS

22/05/2006

Abstract

In the works [1, 2, 3] we examined and criticized the Problem of the two bodies, fundamental argument of the Modern Celestial Mechanics. In these works we posed in evidence the following salient points.

1. We observed that from the Universal Gravitation Law by the everlasting Newton we deduce, differently from what Galileo affirmed, the formalities the graves fall on the terrestrial surface depend on the mass of the falling body. In fact while Galileo asserts that graves only passively suffer gravitational action of the Earth; Newton extends to all the bodies of the cosmos a proper gravitational power, the graves, actively and themselves, attract, proportionally to their mass, the Earth toward themselves so the time of fall of a body on the terrestrial surface is also function of the mass that falls. In fact from the solution of Newton's Problem of the two bodies (and so by the actual Classic Celestial Mechanics) we easly draw [1,2,3] that the acceleration of a grave is given by the formula

$$g = \frac{GM}{R^2}\left(1+\frac{m}{M}\right) = g_o\left(1+\frac{m}{M}\right),$$

where m, M and R are respectively the masses of the grave, of the Earth and the terrestrial ray. So from the said formula we deduce that only when the relation m/M is completely negligible we obtain, as a case limit, Galileo's fall law. On the terrestrial surface the relation m/M, for a ton grave, is of the order of 1×10^{-22}, so the differences of acceleration among the various graves still escape to any experimental control, even if executed with the most refined modern instruments.

2. So if, actually, is still impossible to verify what the actual Classic Celestial Mechanics foreseen on the terrestrial surface, in our solar system things are quite different. In that case the said relation goes from 1.65×10^{-7} ([1]), for the couple Sun-Mercury, to 9.54×10^{-4}, in the case of the binary system Sun-Jupiter ([2]). This involves that the motion to the sun engraved by the mass of a generic planet is not quite negligible, while, on the other hand, <u>in the actual theories of the planetary perturbations, both in the two bodies and in the n bodies problem, the Sun is always considered absolutely fixed in comparison to the *fixed stars*.</u> In fact the relative motion measured between the point γ and the Sun, about 50" sessagesimal arch a year, is nowadays still entirely attributed only to the point γ motion, by justifying (or trying to do it) all by lunisolar precession phenomenon only and thinking about the Sun to be completely fixed, despite the non-negligible actions of shifting or recoil he suffers from the various planets and particularly from Jupiter. In conclusion we will show that the missing prevision of the Celestial Mechanics about the advancement of Mercury's perihelia is exclusively given by the exquisitely theoretical fact that, said theory, in some precise moments of the calculation algorithm development we are going to individualize, unconsciously it implies, contrarily to its native presuppositions, the absolute fixity of the Sun.

But this implied free fixity of the Sun that, in the specific case of Mercury, costs to C.M. a prevision error of just 44'' a century, it shakes and strongly upsets it in the

1 By now on, mind this number.
2 For the system Earth-Moon it even climbs $m/M=1.23 \times 10^{-2}$!

explanation of the grandiose phenomenon of the lunisolar precession, explanation in which, once more, it tries to attribute to the gyroscopic phenomenas only the relative motion of 50'' a year recorded between the Sun and the point γ, to the sole motion of that one, this explanation, for other non-suspect aspects, already strongly contested at its time by scientists like Bernoulli, Eulero and d'Alambert, in '700. According to the quoted Authors, Newton's explanation only would not succeed in justifying the whole observed phenomenon, but these researchers and illustrious scientists didn't advance other hypotheses or effects that would have been able to fill this deficiency that, according to the them, it would explain only about 30'' a year and not a century. In addiction, the actual acquirements about the terrestrial geoid, constituted by a thin crust that floats on an incandescent magma, also sets in strong discussion the value of the moments of inactivity of the geoid itself, obviously attributed as ad hoc [2], so Newton's explanation would go down to about ten seconds a year [2]. To this big and unloadable deficiency of Newton's justification, as shown in this script, can die the deceasing that instead the Sun suffers from all the various planets, motion unduly annulled by the Celestial Mechanics. So it's finally found the law with which the move of the Sun toward the point γ should vary and it also succeeds in theoretically appraising the annual variation that suffers that one nowadays we commonly call lunisolar precession. The following graph gives the course of this phenomenon (Fig. 30)

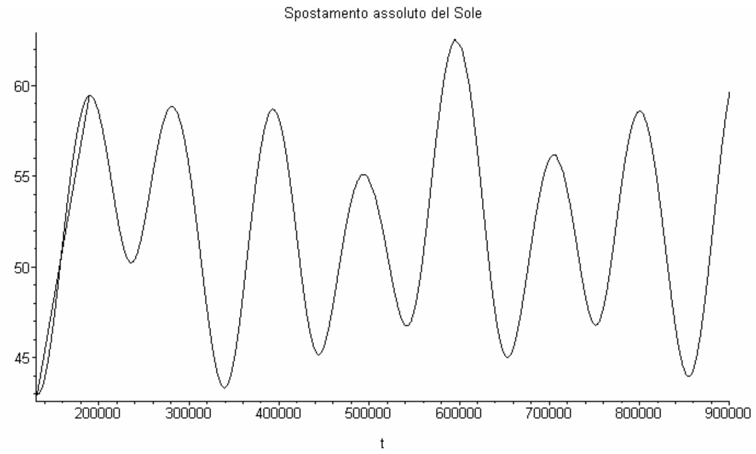

Fig. 30

on the abscissa the time is represented in years and on the ordinates the value of the move of the Sun toward the point γ given by the contribution of all the planets, included the negligible 44'' a century, because of the recoil action of the Sun caused by Mercury. Theoretical variation amounts at ±0.00027 sessagesimal arch seconds a year. The experimental one, by Newcomb, at the beginning of last century it amounted at 0.000222. <u>Instead this variation is totally incomprehensible and unjustifiable with the actual interpretation, absolutely gyroscopic.</u>

So we have the impression, only because we want to avoid to say we're certain, that if we remove the unbearable and inopportune theorist fixity of the Sun from C.M.'s perturbations theory, that also involves some substantial afterthoughts on the sidereal time, the theory of the insuperable Englishman will give again <u>unexpected, positive and strong surprises</u>.

1 – On the Absolute Relative Principia

Let's consider a binary gravitational system constituted by two identical masses **M** gravitating one around the other on a circular orbit of ray r and diameter d, all represented in Fig. 1. To distinguish them they have different colors.

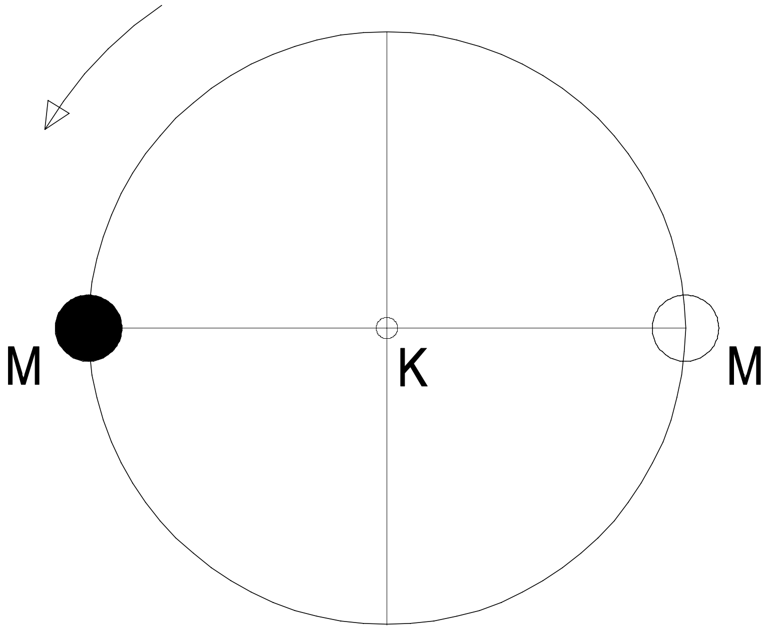

Fig. 1

In this case we say they rotate with the same speed around the barycentre K of the two masses. But, as it's simple to see, we can, in equivalent way, also to affirm that they, mutually rotates, or they revolutionizes the one around the other. By referring to the Fig. 2, let's do the following reasoning.

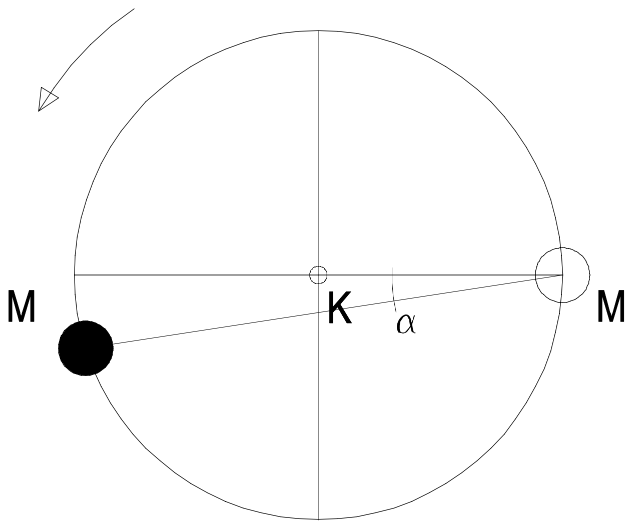

Fig. 2

Let's think about holding for a brief time the white mass of right fixed and to leave free that black one of left. This last one, in the said time, will bring itself in the new position pointed out by the sketch. So the *instant* conjunction of the two masses will describe the angle α,

representing the little revolution the mass on the left will have described around the centre of the mass on the right.
Subsequently, we hold fixed the mass of the left in its new assumed position and we free the one on the right. This last one, now orbiting around the black mass, it will assume the position we draw from the Fig. 3.

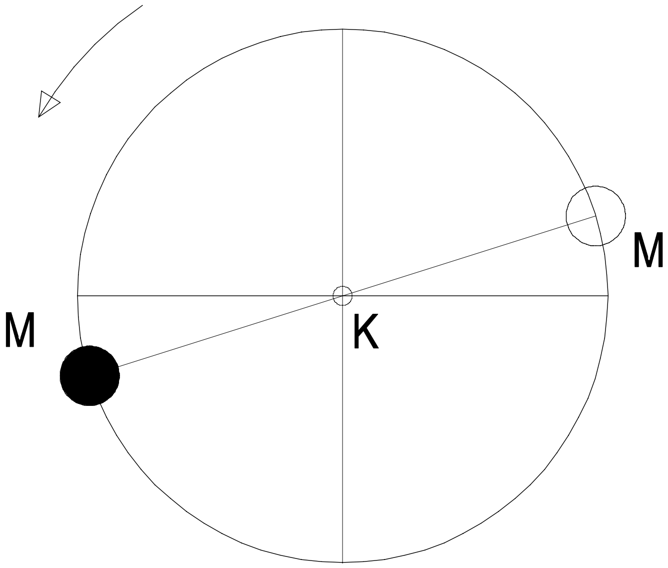

Fig. 3

As we can see from this sketch, the two masses, by revolutionizing one around the other, let the segment that instantly connects their centers always pass for the barycentre K of the two masses. It's possible to determine the general angle of revolution α they describe *the one around the other* in a complete rotation of the two masses.

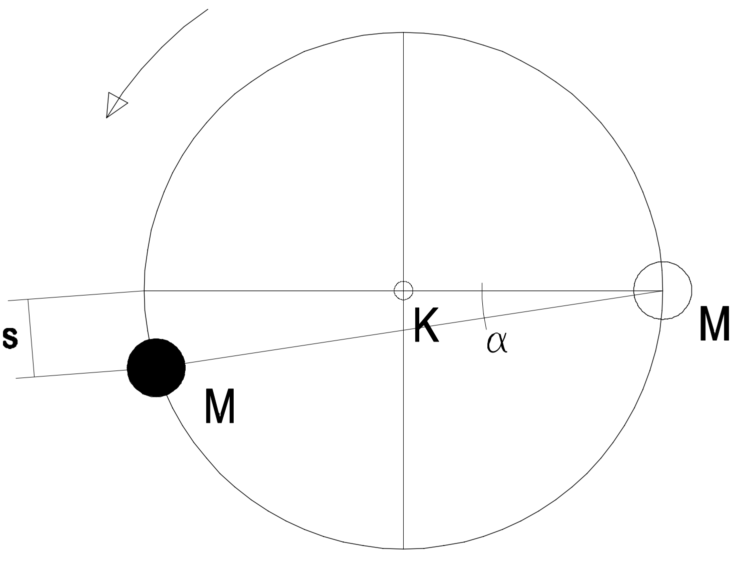

Fig. 4

From the Fig. 4 we have

$$\alpha^{rad} = \frac{s}{d} = \frac{s}{2r} = \frac{2\pi r}{2r} = \pi \iff \alpha^{o} = 180^{o} \tag{1.1}$$

so a mass will orbit around the other of 180°, the second one of other 180° so the general sum is equal to the entire angle of 360°. So, while the two masses will orbit around K exactly of a 360° angle, the same ones will orbit one around the other of a 180° angle.
Let's make this result more intuitive. Let's consider the Fig. 5. In it are represented

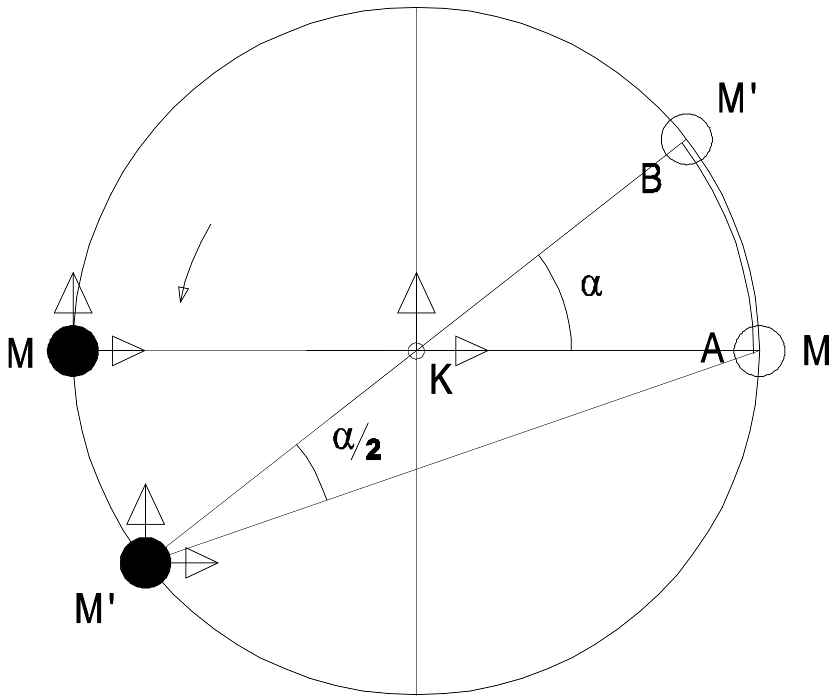

Fig. 5

reference systems anchored to K and to the mass on the left. For the observer K the white mass on the right will describe, in a determined time Δt, the angle α. In fact to the time t=0 he sees the white mass in A and then, passed the Δt, he sees it in B.

The observer anchored to the black mass will equally see, in the same instants of time, the white mass first in A and then in B, but this mass, in its reference system, will only have described an angle equal to α/2 (it's about two angles, the one in the centre (α) and the other one at the circumference (α/2), which insist on the same arch MM').

So, while for the observer K the white mass will have described a complete 360° angle during the time T, at this same time the same mass, by the observer anchored to the black mass M, it will only have described a 180° angle. In fact, hand by hand the black mass M passes from its primitive position M to the next one M', even the primitive position A of the white mass little by little changes its position in the reference system anchored to the black mass. Instead, in the system K the primitive position A of the white mass remains unchanged.

Now let's examine the general case of a central mass equal to M and a peripheral one of mass m (Fig. 6).

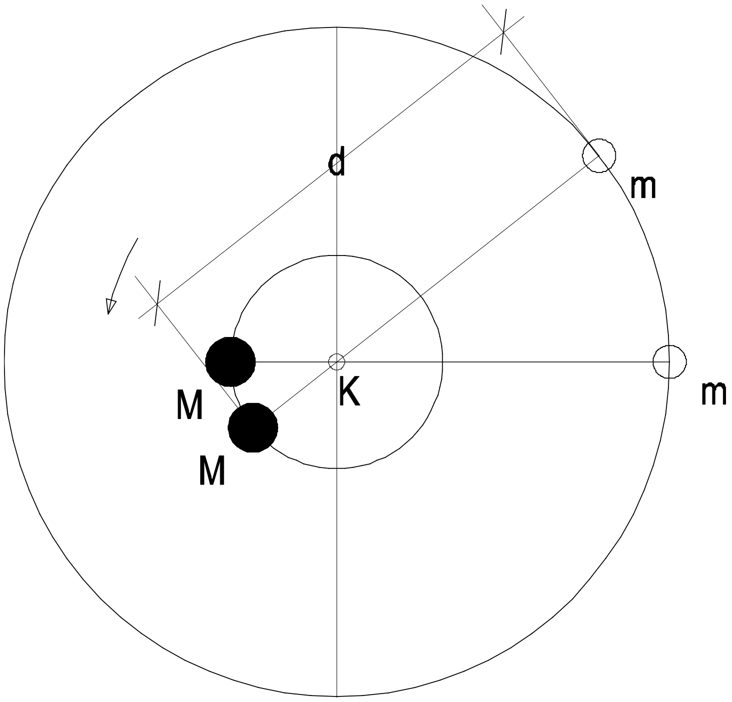

Fig. 6

It’s know that the distance M-K is given by the relation

$$\overline{M-K} = \frac{d}{M+m} m \qquad (1.2)$$

and the distance K-m is

$$\overline{K-m} = \frac{d}{M+m} M \ . \qquad (1.3)$$

in that case, referring the preceding reasoning, we have that the mass M, moving on the circumference of ray M-K, it will orbit around m of angle equal to

$$\alpha_M^{rad} = 2\pi \frac{d}{M+m} m \frac{1}{d} = 2\pi \frac{m}{M+m} \Leftrightarrow \alpha_M^{o} = 360° \frac{m}{M+m} \qquad (1.4)$$

while the mass m, crossing the circumference of ray K-m, it will orbit around M of angle

$$\alpha_m^{rad} = 2\pi \frac{d}{M+m} M \frac{1}{d} = 2\pi \frac{M}{M+m} \Leftrightarrow \alpha_m^{o} = 360° \frac{M}{M+m} . \qquad (1.5)$$

in conclusion the general angle of revolution is always equal to

$$\alpha = \alpha_M^{\circ} + \alpha_m^{\circ} = 360° \left(\frac{m}{M+m} + \frac{M}{M+m} \right) = 360° . \qquad (1.6)$$

in the case already examined to the beginning, in which the two orbiting masses are equal and uniform and equal to M, from the (1.4) and the (1.5) we obtain again the result (1.1).

And always from the (1.4) and the (1.5) we also see that, little by little the mass m decreases until its complete annulment, the principal mass revolution M around the secondary one reduces more and more until it completely annuls. The principal mass revolution around the secondary one, when this last one is completely negligible respect to M, in basis to the (1.4), it becomes null. In fact, in that case, the central mass is absolutely fixed and so its shifting or movement from the starting point is rigorously null. In this last case only, the reference system anchored to the central mass is rigorously inertial and the entire 360° angle is all to be attributed to the peripheral mass only.

Reassuming we can say that if the two masses are identical (and therefore the barycentre K is exactly to the center of the segment that connects them) a mass will revolutionize around the other of 180°. Hand by hand the mass of one of them tends to annul (and so by a consequence the barycentre K moves more and more toward the center of the central mass M), there’s the increase of the revolution angle of the mass m around the principal one and there’s the decrease of the revolution of this last one around the secondary mass m. When m gets null, it’s only this last one to make a complete revolution of 360° around the principal one, because M doesn’t move at all from its native position.

So we have to do this first conclusion. If we simply consider the two reference systems anchored to the two masses M and m, we could say, as commonly and currently there’s the belief, that any of the two observers could legitimately be considered fixed and to attribute the whole motion of revolution to the other mass and vice versa and this to neglect the different accelerations to which the same ones are submitted.

But, as we have seen, it’s just the masses that allow us to establish instead how much relative or apparent revolution is to be attributed to the one and how much to the other one. On the other hand to admit that, indifferently, one of the two observers could consider himself fixed and to attribute to the other one all the motion is equal to deny, besides, the fundamental Principia of physics saying that the barycentre K is always gifted of quiet or rectilinear and uniform motion.

In fact the said barycentre, in comparison to the observer that wanted to consider himself firm, it would be endowed with accelerated motion, in how much K would describe a circumference around the observer. It’s opportune to underline that, in the cases now examined, we are in presence of non inertial systems of reference, which warn therefore, independently from what we visually noticed, different accelerations. Instead, as we are about to see in the Problem of the two Bodies, as currently formulated in Celestial Mechanics and from Newton, the passage from the inertial system anchored to the barycentre K to that non inertial one loyal to the central mass happens considering both these reference systems completely and absolutely inertial!

This is a very important point that must be clearly underlined and confirmed.

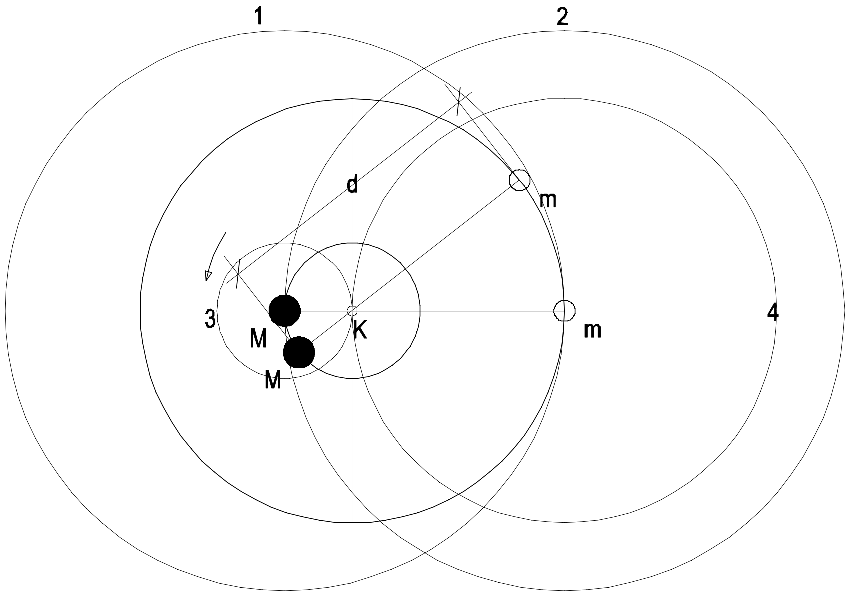

Fig. 7

According to the usual, undisputed and traditional procedure, the observer loyal to M (Fig. 7), can legitimately consider himself firm and to attribute all his motion that has in comparison to K, to the secondary mass m. So this last one describes, in the reference system anchored to M, the orbit 1 and, contemporarily, the barycentre K, the orbit 3.

As many and with the same identical legitimacy, the observer anchored to m can do. In this last case the orbit described by M is that suitable with the number 2 and the barycentre K it would describe, this time, the orbit 4. So it’s therefore clear that this way of progress violates the certain Principia saying that the centre of masses is in quiet or runs a uniform motion. Instead if, in base to what we’ve just mentioned, we consider the two masses revolutionizing the one around the other of the quantities given by the (1.4) and (1.5), we have the rigorous respect of the said Principia. In fact, see the Fig. 8, if, adjusting the

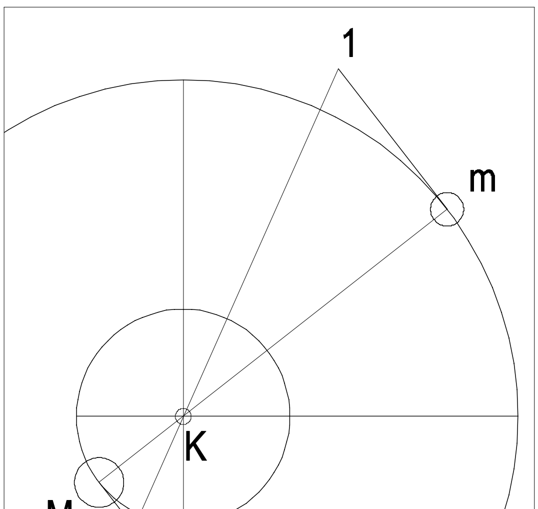

Fig. 8

the said reciprocal revolutions of the two masses, let's point out by the tract M-2 the revolution of M around m and by m-1 the revolution of m around M, connecting the two points 1-2, the said segment will always intercept the barycentre K.

What we noticed previously can be synthesized in easy sketches (see the Fig. 9 and 10) showing the revolution angle of the mass m around the central mass M when m is little in comparison to M but it's not entirely negligible.
The Fig. 9 shows a big part of the apparent revolution and it's to be attributed to the real revolution that the secondary mass m effectuate around the principal one.

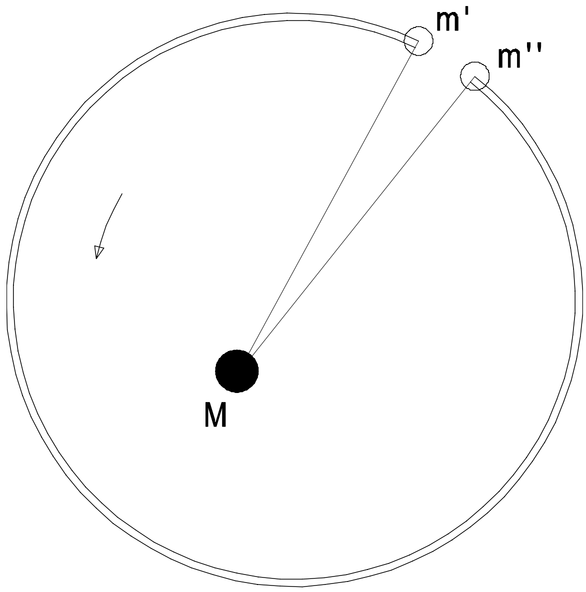

Fig. 9

Instead the fig. 10 show that small real part of the whole apparent revolution describing the mass M around the secondary mass m, represented by the little arch M-M'.

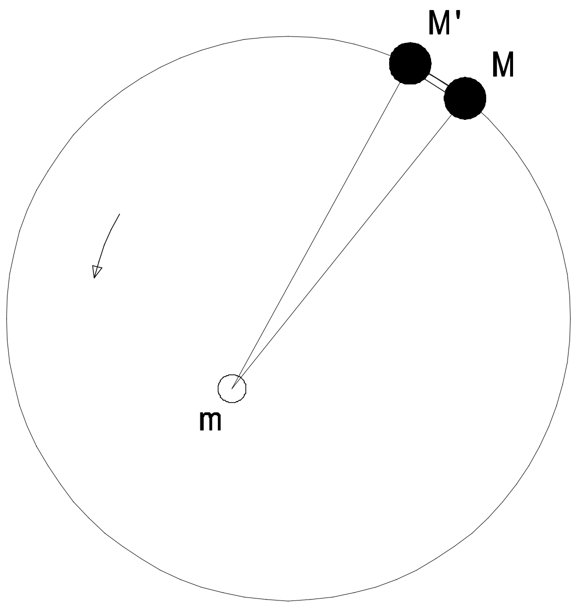

Fig. 10

Let’s consider, at last, the case the two masses M and m fall the one toward the other, along their conjunction (Fig. 11).

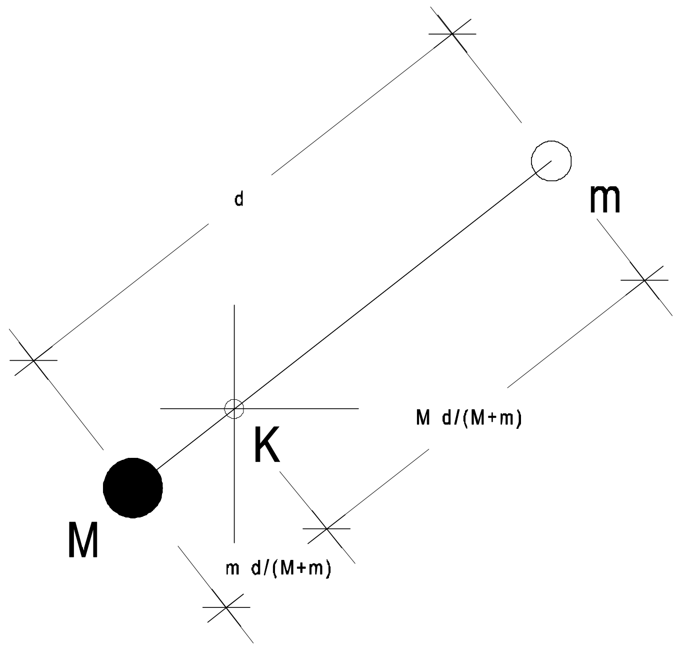

Fig. 11

It’s evident, since they will reach the point K in the same instant, that the mass m will fall toward the central one crossing, the tract K-m, of the whole distance d, and vice versa, the central mass M will fall toward m crossing, of the distance d, only the line M-K and this in perfect simile with what we’ve already seen in the case of the mutual revolutions first considered. In fact to the distance **d** corresponds, in the case of the orbiting masses, to the entire **360°** angle. It’s also easy to control that, in the reference system K, the product of the masses for the path done by these ones, and that’s both in the case of the fall and in the case of the revolutions of the one around the other, it’s constant. In fact in the case of the vertical fall we have

$$M\frac{d}{M+m}m = m\frac{d}{M+m}M\ , \tag{1.7}$$

and in the second one

$$M\frac{2\pi d}{M+m}m = m\frac{2\pi d}{M+m}M\ . \tag{1.8}$$

if we divide the two members of the previous equations for the time of fall or revolution T (measured in the system K) we have

$$M\,V_M = mV_m\,. \tag{1.9}$$

this is a condition even respected from Newton’s theory, as we will see.
In addiction, the angular speed of the two masses, always in the system K it results the same one. Instead, for two reference systems anchored to the two masses we would consider absolutely equivalent among them (as it is well known, they are equivalent among them the inertial systems of reference only), the mass m would have crossed the whole distance d and vice versa.

It is necessary to observe that the time T, for the system K, is both the time that it employs the mass m to cross the whole circumference of ray K-m, and the time that employs the mass M to completely describe the circumference of ray K-M. For the reference system anchored to the central mass M, the peripheral mass m, in the universal time T, has completed instead an angle of reciprocal revolution equal to

$$\alpha_m^o = 360° \frac{M}{M+m}, \qquad (1.10)$$

around M, angle that, as it is seen by the (1.10), it is almost equal to the whole circle angle, generally seen the littleness of m in comparison to M. At the same time T, the principal mass M has completed, around the secondary one m, the arch

$$\alpha_M^o = 360° \frac{m}{M+m} \qquad (1.11)$$

and therefore adding the two angles, we have the whole circle angle.

Therefore the difference among the whole circle angle and the value given by the (1.10) must be attributed to the rotation that the central mass effectuates around m, always at the same time T and therefore it’s to be attributed to the real movement of the central mass around the secondary one.

In base to what we’ve previously noticed, we deduce that the angular speed of the reciprocal revolutions of a mass around the other one are well different. In fact we have the mass m to revolutionize around M with an angular speed

$$\omega_{M\,m}^o = \frac{360°}{T} \frac{M}{M+m} \qquad (1.12)$$

while we will have

$$\omega_{m\,M}^o = \frac{360°}{T} \frac{m}{M+m}. \qquad (1.13)$$

the sum of these two angular speeds

$$\frac{360°}{T} \frac{M}{M+m} + \frac{360°}{T} \frac{m}{M+m} = \frac{360°}{T} = \omega \qquad (1.14)$$

it is equal to the angular speed of the two masses in comparison to the barycentre K.

It’s opportune to recapitulate what we have already said. Usually there’s the belief that it’s possible, according to the reference systems adopted, to attribute the whole apparent or related motion to the mass that doesn't belong to the chosen reference system. That’s what is done in the actual Celestial Mechanics. Let’s examine, apropos, the Fig. 12.

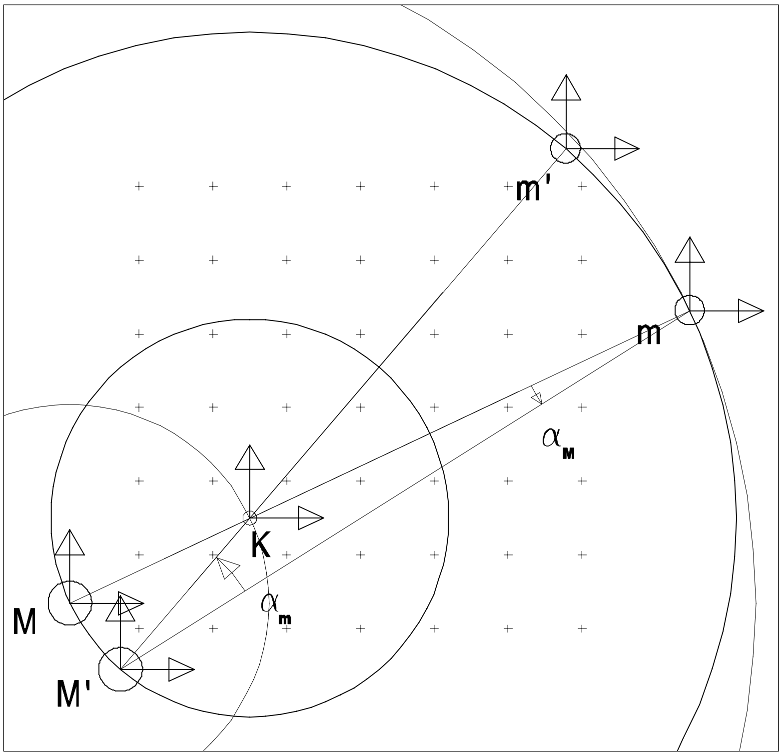

Fig. 12

As we've already said, even if it is admitted that the two bodies of masses M and m orbit around the common barycentre K, when we pass to the reference systems anchored to M and m, it is admitted that they can absolutely consider legitimately themselves firm, even in comparison to the fixed stars, represented in the figure from as many dots, and it is attributed the whole noticed movement to the other mass.

As already observed, by operating this way, in the case reference system anchored to the mass M, the secondary mass m would not describe the circumference of ray K-m anymore on the contrary that greater one , of ray M-m. In addiction, the barycentre K, doesn't remain in quiet anymore (or endowed with rectilinear and uniform motion in comparison to the fixed stars), but it describes, in accelerated motion, the circumference of ray M-K and this contrarily to the known and incontrovertible physical Principia.

Because the barycentre K is only a theoretical point, astronomically not reliable, it's instead possible to make the following reconstruction of the phenomenon. In the hypothesis that at the time t=0 the two masses occupy the positions M and m, to find again their positions in the little time Δt, making a pivot around m, the segment M-m=d of the angle can be rotated α_M (1.13). By this operation, because of the departed infinitesimal time, the mass M goes to place in M'. Subsequently, making a pivot around M', rotating therefore the segment M'-m of the angle α_m (1.14), the secondary mass m goes to the new position m' assumed after the said little time.

By these two operations it's possible to find again the positions of the two masses and in addiction <u>the straight line that instantly connects them always passes for the barycentre K.</u>

So it's therefore to observe that the simple value of the masses allows to eliminate entirely the ambiguities of an apparent and deceptive relativism it's found again in the celestial

Newtonian mechanics, despite the presence of indisputable and different accelerations and the efforts Newton himself.

And we must remember the criticisms that He suffered both from the archbishop George Berkeley, his contemporary, that, subsequently, from the acute E. Mach [4] just about the absolute motions and the relative or deceptive motions. Newton, in his everlasting work the **Principia**, just about reading it, to discern and to draw the laws of the physics from the world of the common appearances (for instance, that one of Tolomeo), he makes a famous experiment. He takes a bucket full of water and hangs it with a thread to the ceiling of his room and engraves to it a rotation around the axle of suspension.

After a few he observes that the water's surface, first plain, then it becomes concave, going up again toward the edges of the vase and going down toward the centre. Then he affirms that the bucket is endowed with an absolute motion of rotation (he would have been able to add the sentence *in comparison to the fixed stars*, but he didn't do it!). Because of this deduction George Berkeley attacked him, sustaining that an observer that rotates with the bucket can be considered firm and he can see the fixed stars rotating around him in the opposite sense. It was therefore entirely arbitrary the reasoning of Newton. In effects Berkeley sustains that an observer that rotates around his vertical axle, despite his inevitable dizziness and the fact that his arms from vertical that were, they have the tendency to horizontally get, he can quietly consider immovable and to think that the whole rest is turning around him!

Subsequently the admirable Mach added [4], strengthening the criticism of Berkeley, that the stars are not simple bright dots, but they have considerable masses. Later, with Einstein, we have that if the stars would rotate with the same speed of the bucket and this last was firm, in the same way the water in the bucket would assume the configuration noticed by Newton and therefore equally the observer anchored to the bucket would warn the same dizziness and his arms would be horizontally stretched.

We are really sorry to notice that the concordant version of Berkeley, Mach and Einstein (BME), founding itself on pure deceptive facts or also appearances, it also violently bumps against another elementary physical Principia. In fact, when Newton sets in rotation the bucket, for the Action and Reaction Principia (III° Principia of Dynamics, formulated from Newton himself), He can't do anything else than to apply an equal and contrary action to the floor of his laboratory in a way he cannot do anything else than to engrave to the floor of the laboratory an equal and contrary action. So if we indicate by I_m and I_M the inertial moments of the bucket and of the laboratory by ω_m and by ω_M the respective angular speeds of the one and the other, acquired after the rotary impulse given by the operator, we have a formula analogous to the (1.9) and that is

$$I_m \omega_m = I_M \omega_M \ , \tag{1.15}$$

This Principia was broadly verified and well known even to Cartesio, although in a different form. From this it derives that, with Newton, we succeed in establishing, once more, than it will rotate the laboratory and than the bucket. Since the inertial moment of the bucket is infinitesimal in comparison to that one of the fixed stars (or, generally, to the walls of the laboratory) we can be well certain that, practically, almost all the *apparent and deceptive* movement detectable between the bucket and the fixed stars is to be attributed in a wide part to the bucket and, partly infinitesimal, to the stars of Berkeley and Mach and therefore Newton was and is still at 99.999 % right. So not only we succeed in establishing who and how it rotates but we exactly determine the entity of the reciprocal rotations. But we have to add that if it is experimented with two or more buckets contemporarily, endowed with different and contrary angular speeds, what will this enormous heap of stars do by (BME) who will it satisfy? Instead, also in this case a little more complicated, contrarily to the impossibility of the answer to this question from BME, once again it's possible to establish, always in base to the III° Principia of Dynamics, what the resulting force will be of all of these motions and therefore it will succeed in determining, with absolute precision, how and how much, and in what a sense will rotate the various bodies summoned.

And in addiction we add that if, thing not negligible, we would like to adorn the said stellar heap with an angular speed equal to that one of the bucket, this last one, always for the III° Principia of Dynamics, it would come to obviously possess an unbelievable speed. It is the same known fact of the ball, of mass m, shot by a gun (of mass M). We know that for these two bodies it is still valid the (1.9) (Quantity of Motion Conservation Principia) and that is

$$M\, v_M = mV_m \qquad (1.16)$$

and therefore we succeed in engraving a great speed to the proper bullet and only for the littleness of its mass in comparison to that one of the gun. If we affirmed that the battered observer anchored to the ball of gun can legitimately consider himself firm and to attribute the whole measured relative speed between the two masses to the gun only, it should be concluded that the quantity of explosive position is enormously greater of the effective one and so to deny, once more, the validity of action and reaction Principia inherent in the (1.16).

It is Mach that said the world is given in an one time only and all together, it's then about up to decompose, and to reduce a complex phenomenon to clear and intelligible principia and instead, reasoning this way, he affirms that it comes first the apparent and deceptive kinematics and then the dynamics. Certainly, an astronaut in a rocket that accelerates can have the illusion to be firm while the whole universe accelerates in the opposite sense, but this is only a pure impression also contrasting with the economy principia of the physical phenomenas because, to accelerate in that measure the whole universe, as the purest relativist would it to be possible, it would be necessary applying to it a propeller of an unheard power.

Neither, to support this thesis, the Principia of Equivalence can be invoked, according to which the said astronaut cannot effectuate any experiment that allows him to establish if he is subject to some inertial or gravitational strengths, seen the Galilei's fall of the graves law. In fact if the astronaut lets some bodies of different masses to fall toward the fund of his spaceship they effectively will exactly reach in the same instant the fund of the missile and this independently on their masses.

A feather, a hammer and a ball of gun would exactly reach the fund of the spaceship in the same proper instant and only because it's not these bodies falling toward the fund but it's this last only, accelerating toward the said bodies (these *falling* bodies, once abandoned the hand of the astronaut, according to the I° Principia of dynamics, they persevere in their state of quiet or rectilinear and uniform motion saving the speed they had during their release).

Instead, in a gravitational field things are well different: Galilei's fall law [2], as we will have the opportunity to see even in this script, it has been revisited by Newton with the introduction in it of the falling masses (Problem of the two Bodies) and therefore the said astronaut cannot confuse the inertial effects, to which he is submitted with those gravitational ones.

And once again the masses allow us to establish how much share of the relative, apparent or deceptive motion must be attributed to the one or to the other reference system. So we have some relative absolute motions or absolutely relative motions. On the other hand the purpose of the physics should be to interpret the common appearances according to the applicable Principias in every case and therefore universal.

After said this, let's translate in formulas the preceding concepts, reassumed and synthesized in the following Fig. 13 .

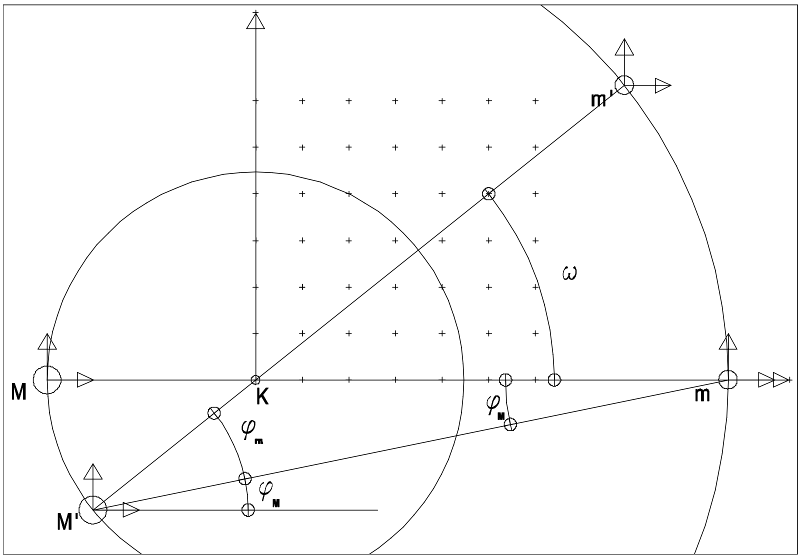

Fig. 13

The coordinates of the mass m, in comparison to the fixed system K, to the time t=0, they are given by the equations

$$x_{Km} = \frac{Md}{M+m}\cos(\omega t) = \frac{Md}{M+m}$$
$$y_{Km} = \frac{Md}{M+m}\sin(\omega t) = 0 \qquad (1.17)$$

where

$$\omega = \frac{2\pi}{T}. \qquad (1.18)$$

the coordinates of M, always in the said system, are

$$x_{KM} = \frac{md}{M+m}\cos(\omega t + \pi) = -\frac{md}{M+m}$$
$$y_{KM} = \frac{md}{M+m}\sin(\omega t + \pi) = 0 \qquad (1.19)$$

the coordinates of **M'**, in the system anchored to **m**, after a little time t, are

$$x_{mM'} = d\cos(\phi_M t + \pi)$$
$$y_{mM'} = d\sin(\phi_M t + \pi) \qquad (1.20)$$

where

$$\phi_M = \frac{m}{M+m}\frac{2\pi}{T} = \frac{m}{M+m}\omega\,. \qquad (1.21)$$

the coordinates of **m'**, in the system anchored to **M'**, are

$$x_{M'm'} = d\cos(\phi_M t + \phi_m t)$$
$$y_{M'm'} = d\sin(\phi_M t + \phi_m t) \qquad (1.22)$$

where

$$\phi_m = \frac{M}{M+m}\frac{2\pi}{T} = \frac{M}{M+m}\omega \qquad (1.23)$$

from the previous relations we have that the (1.22) become

$$x_{M'm'} = d\cos(\phi_M t + \phi_m t) = d\cos(\omega t)$$
$$y_{M'm'} = d\sin(\phi_M t + \phi_m t) = d\sin(\omega t) \qquad (1.24)$$

from the (1.24) we see how, in the mobile reference system M', through the reciprocal revolutions that the one mass makes around the other, to succeed in determining both the position of the mass m' and of M' that it results to coincide with that one related to the system anchored to K.

We can also see the angular speed ω the two masses posses in the system anchored to the fixed stars (or to K) both the sum of the angular speeds of reciprocal revolution of the two masses the one around the other and that is

$$\omega = \phi_m + \phi_M\,, \qquad (1.25)$$

in fact

$$\omega = \frac{2\pi}{T} \equiv \frac{360°}{T} = \frac{360°}{T}\frac{M}{M+m} + \frac{360°}{T}\frac{m}{M+m} = \frac{360°}{T} \equiv \frac{2\pi}{T}\,. \qquad (1.26)$$

The following figures determine the position of M' and m' using the reference systems anchored to them and the preceding formulas. For the Fig. 14

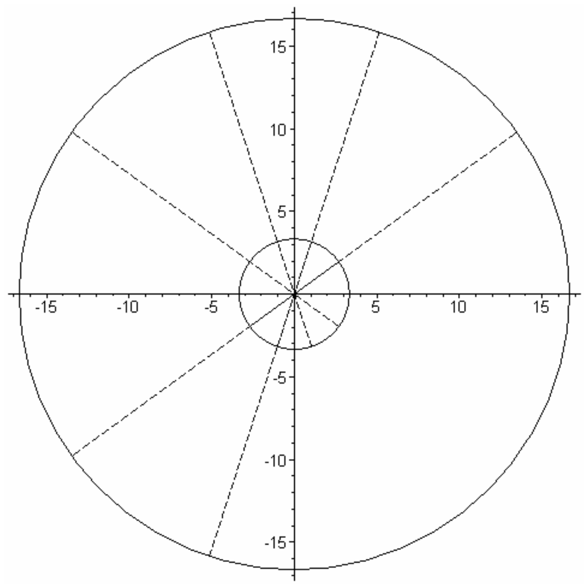

Fig. 14

It has been used a relationship of the masses equal to m/M=0.2. In it, it's clearly seen the orbit crossed by the central mass and the conjunctions, the two masses always pass for the barycentre K. The Fig. 15 is obtained with a relationship

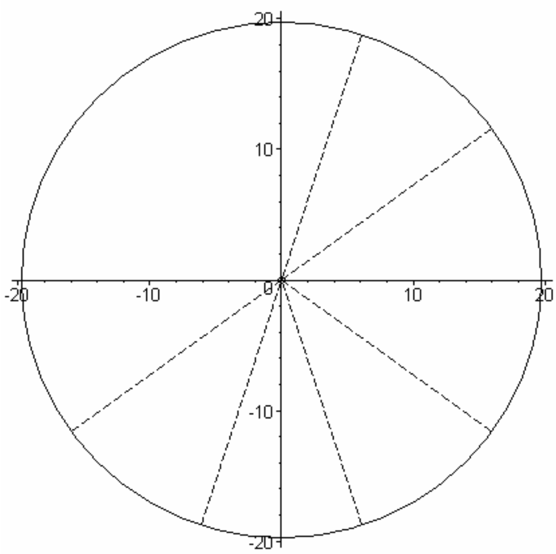

Fig. 15

m/M=1/81. It's the case of the system Earth-moon, in which it is the maximum that can be found in our solar system; in fact, for the couple Sun-Jupiter, the said relationship is notably inferior and equal to about m/M=1/1000. And, as it is possible to se by this figure, the bracelet described by the central mass is imperceptible <u>but not totally negligible</u>.

Until here we have made some pure and simple considerations of exclusive physical and geometric character. As we will see soon, in base to the theory of Newton, the angles of revolution of the masses M and m, in the Problem of the two Bodies, given by the formulas (1.4) and (1.5), they will instead be halved. <u>To see now what we are talking about</u>, let's calculate with these simple and immediate relationships the angle of revolution the Sun effectuates **around** Mercury in a period of 88 days (Mercury's revolution period around the

Sun), without the use of any gravitational theory, unless for the value today attributed to the masses in game, in the System [c.g.s]. In base to the (1.4) it is had

$$\alpha^{o}_{S_{88}} = 360° \frac{m}{M+m} = 360° \frac{0.3285\times10^{27}}{1989\times10^{30}+0.3285\times10^{27}} = 5.94457\times10^{-5} \quad (1.27)$$

and so an angle in sessagesimal seconds equal to

$$\alpha^{"}_{S_{88gg}} = 5.94\times10^{-5}\times3600 = 0.214045"\,. \quad (1.28)$$

in a hundred years we'll have an arch of revolution of the Sun around Mercury of

$$0.21":88^{g} = \alpha^{"}_{S_{100anni}} : 36525^{g} \quad (1.29)$$

from which

$$\alpha^{"}_{S_{100anni}} = 88"\,. \quad (1.30)$$

let's observe, for the time, this angle of reciprocal revolution is practically the double one of the inexplicable advancement of Mercury's perihelia (43.5 "). If besides the Sun effectuates, in one hundred years, an arch of 88 ", to effectuate a whole revolution **around** Mercury it will employ a time equal to

$$88:100 = \left(360^{o}\times3600\right):T_{SM} \quad (1.31)$$

from which

$$T_{SM} \simeq 1\,472\,727 \;\; anni\,. \quad (1.32)$$

Making the same calculations for every planet of the solar system we have the results of the following chart (1.33).

Pianeta	$\frac{m}{M+m}$	*Periodo anni*	*Arco solare in cento anni*	T_{SP} *Rivoluzione solare di* 360° *Anni platonici*
Mercurio	1.65×10^{-7}	0.24	88.84	1 458 802
Venere	2.45×10^{-6}	0.62	515.94	251 192
Terra	3.00×10^{-6}	1.00	389.39	332 828
Marte	3.24×10^{-7}	1.88	22.34	5 801 253
Giove	9.53×10^{-4}	11.86	10 412.66	12 446
Saturno	2.85×10^{-4}	29.46	1 255.26	103 245
Urano	4.36×10^{-5}	84.01	67.24	1 927 424
Nettuno	5.29×10^{-5}	164.8	41.59	3 116 134
Plutone	4.97×10^{-9}	247.7	0.00	∞

(1.33)

In it, in the fourth column, the arch of revolution of the Sun is brought, expressed in sessagesimal seconds, it describes, around the planet, in one hundred years. In the fifth column it's brought, in years, the time that would employ the Sun to describe the complete orbit of 360° around the planet. We will call this last time platonic related year to the planet considered.

As already anticipated, with the appeal to the theory of Newton, the arch described by the Sun in the fourth column is halved, while, consequently it doubles the platonic year of the relative planet considered. In the case of Jupiter it will be had that the revolution of the Sun around this planet will be of 10412.66 / 2 = 5206 " in one hundred years and therefore equal to 52 " a year, practically almost equal to the value entirely attributed instead nowadays to the lunisolar precession of 50", and in addiction, the period of complete revolution of the Sun around Jupiter would be equal to 12446 x 2 = 24.800 years, even it practically coincident with the known platonic year that is of the 25.700 year-old order. That's why we have called platonic years the times brought in the fifth column.

It would seem to have specified and exhausted the thematic about the passage from a reference system to another but it is not this way. As easily we can see, in Celestial Mechanics, it must be specified and kept well present what follows.

The movements of the celestial bodies all come reported to the fundamental reference system constituted from the scenery of the **fixed** stars, so to be able to affirm that, without them, we would be found in the most complete dark. The ancient astronomers discovered the planets observing that them, in comparison to those ones we call fixed, they were and they are *stars* that quickly stir on the fixed and punctuated scenery of the celestial sky. Not by chance they represented Mercury, also called the mail carrier of the antiquity, with some wings to his feet. In the same way the modern astronomers photograph, in different times, photos of a certain zone of the sky to discover if there is some bright dot that has stirred in comparison to the fixed stars. Then this reference system is of fundamental importance for the C.M..

Instead the deductive scheme according to which the planets rotate around the Sun is one of our logic and physics reconstructions of what we apparently observe, always in comparison to the reference system constituted by the fixed stars. And it is this reference system, that we could momentarily call absolute, that works in a certain sense as a cosmic ether, and that it constitutes the bench of final test of our theoretical reconstructions. Also the tolemaic system, with its elaborate epicycles, succeeded in reconstructing the trajectories described by the planets **on the celestial sky**, but it was only a purely geometric version of the reality and besides a very complex. By the Copernican system instead we have found a reconstruction that better is in accord with those physical principia we succeed in drawing from our daily experiences. Then any gravitational theory owes, in the last analysis, to make the accounts with this reference system, besides, only apparently absolute.

This could seem a superfluous circumstantiation but it is not quite this way. By some simple calculations we have seen that, only in base to the actual value attributed to the masses of the Sun and the planets or better, in base to their relationship, we succeed in calculating some moves of the **solar mass** that, as we will see soon, they are exactly the double ones of those we succeeded in determining with the theory of Newton or in base to the actual Celestial Mechanics. But it is necessary to observe that the modern theories imply to be fixed some stars that aren't this way. In fact today we know that our solar system is in a certain region of our galaxy and all the stars that are in this zone, even our Sun, they stir with a certain law around the center of the said galaxy.

Therefore the motions of the planets that we calculate with the formulas of Newton's theory are **related** to a reference system entirely **arbitrary** and therefore they don't allow us, for instance, to calculate the motion of the Sun and the relative planetary system in comparison to a reference system anchored to the center of our galaxy. And perhaps that's one of the so many motives for which we don't still succeed in bringing back gravity to interactions.

But if we believe that the axle of a gyroscope or the plan in which oscillates the pendulum of Foucault it preserves, in the space, its direction unchanged, for any movement engraved to its support, we could even think those moves brought in the Chart (1.21) are absolute moves of the Sun, while those ones we will calculate by the theory of Newton are related to those we consider to be fixed stars. We will see that the matter is still thinner.

The Fig. 16, that follows, underlines and synthesizes this thematic. The anonymous sheet of paper of the preceding figures on which we have reasoned now is draped of dots (**fixed stars**).

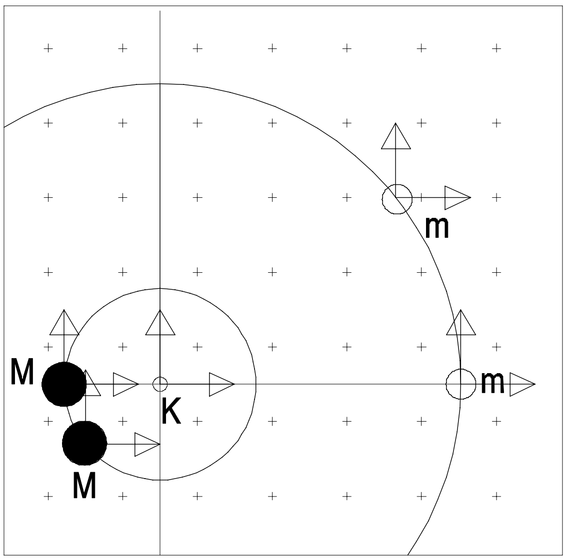

Fig. 16

In it the reference systems anchored to K and the two masses M and m are represented. The system K can be considered loyal to that one of the fixed stars, but it is not astronomically observable so it is therefore a theoretical datum only. This involves that any gravitational theory, using the heliocentric reference system anchored to M, mobile in comparison to that of the fixed stars according to the varying value of the secondary mass, it has to know how to exactly reconstruct the direction in which, in a said instant, it sees the mass m in the reference system anchored to M. So it must be able to individualize what particular **fixed** star, a star that doesn't belong to its reference system, it will be found, in the particular considered instant, perfectly lined up with the points M and m, astronomic greatnesses that are the only to be really observable, unlike the barycentre K.

In truth there's also to be said say that the heliocentric reference system, adopted in astronomy, differs a lot, and this is the fundamental point, from those ones used in theoretical physics because while these last ones have both the origin and the three aces of reference entirely solidali to the body to which they are anchored (it's to think, for instance, to the reference system anchored to a ship and that one related to the firm earth), the astronomic one has the origin in the center of the Sun only, while the aces connect this origin with particular fixed stars (point γ etc.) and therefore external to the solar sphere.

As we will better see soon, this problem list is not leastly faced in the solution of the problem of the two bodies, hinge of the whole consequent theory of the planetary perturbations. In fact, even recognizing that also the Sun rotates around a barycentre K, at the end it is always considered absolutely fixed **even in comparison to the fixed stars!**

We can conclude this part saying that, in the long history of the astronomy, we passed, with great and painful sufferings, and that's not free rhetorical, from the geocentric system to the heliocentric one.

Galilei said the famous sentence **neverthless it moves**, in fighting against a vision dictated by the illusionism of the appearances. Unfortunately we are fallen in another prejudice that finds its reason to be in an acute and ruling scientific heliocentrism.

We have to recognize instead, and it should not be difficult, that if it is true that the Earth **strongly** revolutionizes around the Sun it is also true that the Sun, **even though weakly**, orbits around the Earth. We would be able, paragraphing Galilei, to say **neverthless it moves** but this time is the Sun.

2 - Newton and the problem of the two bodies

As already observed and brought in the works [1,2,3], Newton (and so today's Celestial Mechanics) starts from the presupposition (Fig. 17)

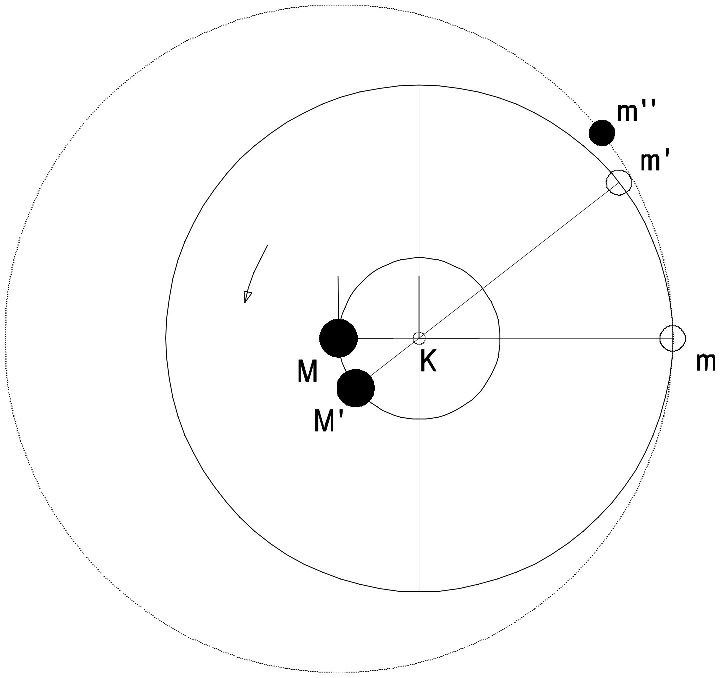

Fig. 17

that the inertial observer anchored to the barycentre K of the two masses, sees applied to them the strength

$$F = G\frac{M\ m}{d^2}. \tag{1.34}$$

this involves that we can write the two obvious equations that equalize the centrifugal strength, that has a body crossing a circumference, to that, centripetal or gravitational, expounded between the two masses and therefore we have

$$M\frac{V_{KM}^2}{\frac{d}{M+m}m} = G\frac{M\ m}{d^2} \tag{1.35}$$

and

$$m\frac{V_{Km}^2}{\frac{d}{M+m}M} = G\frac{M\ m}{d^2}. \tag{1.36}$$

From these equations it is possible to determine the two speeds that have the two masses in the reference system anchored to K. So we have

$$V_{KM} = \sqrt{\frac{G\,m^2}{d\left(M+m\right)}} \tag{1.37}$$

and

$$V_{Km} = \sqrt{\frac{G\,M^2}{d\left(M+m\right)}}\,. \tag{1.38}$$

If, by the (1.37) and (1.38), the period of revolution of the two masses around K is calculated and it is found, in both cases that

$$T = \sqrt{\frac{4\,\pi^2 d^3}{G\,M\left(1+\dfrac{m}{M}\right)}}\,. \tag{1.39}$$

After said this it’s easy to verify that, in the Problem of the two Bodies, it is besides admitted that, in comparison to the system anchored to the principal mass M, the speed of the secondary mass m is given by the simple sum of the preceding ones ([3]) so we have

$$V_{Mm} = \sqrt{\frac{G\,M}{d}\left(1+\frac{m}{M}\right)} \tag{1.40}$$

from which is drawn again that the period of revolution of the mass m around M will be given again by the relationship

$$T = \sqrt{\frac{4\,\pi^2 d^3}{G\,M\left(1+\dfrac{m}{M}\right)}}\,. \tag{1.41}$$

In conclusion, as it will be seen better subsequently, the Sun is considered completely firm, in comparison to the fixed stars and the motion that it has in comparison to them, is

[3] Narrow relativity imposes instead another rule of composition of the speeds. From here a new possibility to relativize the theory of Newton not before having specified fundamental matters that even invest the simple and intuitive galilean relativity. In fact, as it is easy to imagine, also in this last case, the masses have a fundamental role because highly selective. If Galilei had known the mass m of the ship and that one M of the firm earth (or better, their inertial moments), without any experiment, but only in base to the Action and Reaction Principia and to the quantity of motion Conservation, he would have been able to calculate with absolute precision how much of the measured relative speed between the ship and the dry land he would have had to attribute to the one and the other not to invalidate incontestable Principia. But He didn't have any certainty of it. From the considerations already done it achieves that the formulas of passage from a system of inertial reference to the other one should contain the value of the interactive masses, thing that, notoriously, in the galilean transformations entirely misses (and not only in them), denying of fact the existence of the mentioned Principias! It’s therefore clear that it doesn't deal with being able to discern between the quiet and the absolute motion of a body (medieval idea still survived) that, unconsciously it pretends the existence of a cosmic ether, and therefore the presumed existence of a privileged system of reference, on the contrary to establish the shares of the apparent or relative motion that legitimately go instead attributed at the two or more interactive masses and this in respect to mentioned Principias, otherwise it is useless to strive us to conceive them. What therefore the medieval ether exists or less, under this aspect, it doesn't have any importance. Rather, if the said ether was a sort of emanation or physical field of the masses themselves, it would owe, obviously, also to accordingly behave to the laws of these Principia that are, after all, principias of inertial character (state of motion or quiet conservation). From here the Principia of the Absolute Relatives. It’s therefore easy to understand whether to select, among the infinite inertial systems, the one establishing what is happened between the two interactive masses. But where are the masses in the equations of Maxwell and which role would they have in this case?

entirely attributed and mathematically added to that one of the secondary mass, motion that this last one always has in comparison to the fixed stars or in comparison to the reference system anchored to the barycentre of the masses K.

Then the observer anchored to the Sun attributes all of his motion to the secondary mass and is considered completely in quiet. It also could be legitimate that the observer anchored to the Sun can consider him in quiet, but this could be permissible only in absence of fixed stars, to which then, in the last analysis, it is necessary to refer and this to want to neglect completely the undeniable acceleration that he however warns and from which he can never free.

To underline this clear contradiction we make the following reasoning, based on the equations of Newton.

We set on a circular orbit, of ray **d**, a non negligible mass **m'** in comparison to that central one **M**. It then, in base to the Theory of Newton and that is in base to the actual Celestial Mechanics (CM), it must be introduced on the said orbit with the speed given by the (1.40). Instead, in the case in which we wanted to set on the same orbit a negligible mass, such that is m''/M=0, this last one will obviously have to possess, always in accord with the (1.40), a speed of revolution equal to

$$V_{M\,m''} = \sqrt{\frac{G\,M}{d}}\ . \qquad (1.42)$$

Let's bring the results (1.40) and (1.42) on the same graph (Fig. 18).

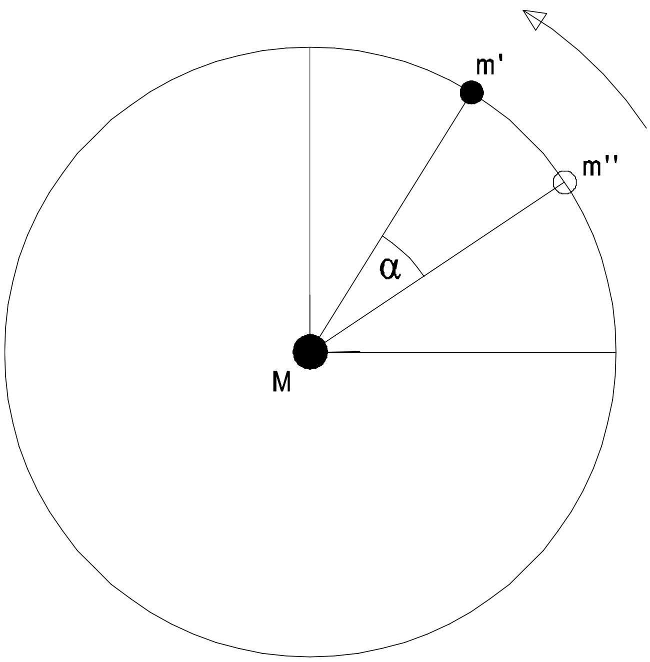

Fig. 18

It results evident, being m' (1.40) faster than m '' (1.42) that, after a complete revolution of the two masses around the Sun, in the hypothesis that they have contemporarily departed from the same point, they will be found in two different positions and so that the relative vector rays will form the angle α of Fig. 12.

The said angle can be calculated. The variation of the orbiting speed in comparison to the variation of the mass m, deriving the (1.40), it is given

$$\frac{dV_{mM}}{dm} \simeq \frac{\Delta V_{mM}}{\Delta m} = \frac{1}{2}\frac{1}{M+m}\sqrt{\frac{GM}{d}\left(1+\frac{m}{M}\right)} \tag{1.43}$$

from which follows

$$\Delta V = \frac{1}{2}\frac{m}{M+m}\sqrt{\frac{GM}{d}\left(1+\frac{m}{M}\right)}. \tag{1.44}$$

Obviously this reasoning is valid until the mass m, although not negligible in comparison to M, would be however small enough. Otherwise, to calculate ΔV, it is necessary to do the simple difference among the values given by the (1.40) for the two values of the masses. By this more speed the mass m', in comparison to the mass m '', in the time T of revolution (1.40), it will have crossed the arch of length

$$s = \frac{1}{2}\frac{m}{M+m}\sqrt{\frac{GM}{d}\left(1+\frac{m}{M}\right)}\sqrt{\frac{4\pi^2 d^3}{GM\left(1+\frac{m}{M}\right)}} = \pi\frac{m}{M+m}d \tag{1.45}$$

and so the angel α of the Fig. 12 will be given by

$$\alpha^{rad} = \frac{s}{d} = \pi\frac{m}{M+m} \Leftrightarrow \alpha^{o} = 180^{o}\frac{m}{M+m}, \tag{1.46}$$

angle that is exactly the half of that one found with the simple previous geometric considerations (see the formula (1.11)). It’s to be noticed, for instance, that if it’s applied the (1.46) for the couple Sun-Mercury is had an angle equal to 44'' a century and that is just the same angle that the classical C.M. would not succeed in explaining.

<u>But is it rigorous what we have just deduced or *the things are quite different?*</u>

In reality, it is easy to become convinced that this angle given by the (1.46) is not the arch that crosses the mass m' in comparison to m '' of more, but it is instead the arc of revolution that the Sun effectuates **around** the secondary mass m', which forces it to move from M to M', dragging, in this rotation, even the orbit that, instant for instant, the secondary mass describes around the Sun, as it is noticed from the Fig. 19.

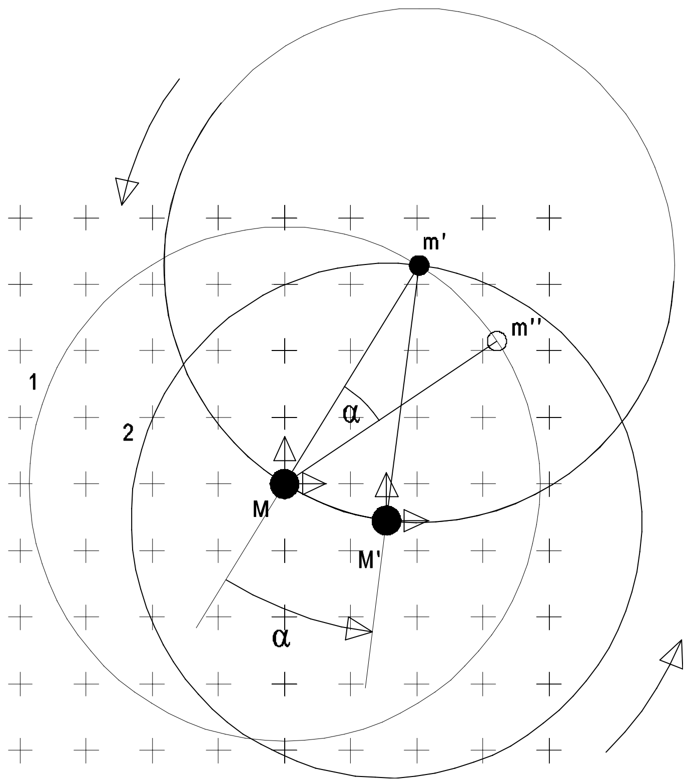

Fig. 19

In fact, whenever the orbiting mass m is entirely negligible in comparison to that one M of the Sun, for what already said previously, this last one, won't stir at all from its native position M that it has in comparison to the fixed stars, (neither, an accelerometer set in the reference system M will report some acceleration), and the orbit described by the secondary mass will exactly be that one suitable with the number 1 of the Fig. 19.

To be noticed that, in this case only, both the *local* reference system anchored to M and that one loyal to the fixed stars, will find the same results; in all of the other cases in which the exemplificable hypothesis is not sustainable anymore saying that the secondary mass is not negligible in comparison to the central one, we owe to attend divergences between the results of the calculations and the experimental comparisons.

In fact, if the secondary mass is not negligible anymore in comparison to the central one we have to recognize that even the mass M orbits around the secondary mass and therefore that apparent and deceptive advancement of the mass m '' in comparison to the mass m', represented by the angle α of Fig. 18, it's instead owed to the real demotion of the central mass M, recoil or real demotion, represented in Fig. 19.

On the other hand the construction of the formula (1.40) has been gotten adding the two speeds that the two masses M and m have in comparison to the fixed system (or inertial one) K (or to the fixed stars) and therefore we should not marvel when that apparent speed in more that the mass m has in comparison to the central one that it has to be attributed to this last only.

The observer anchored to the central mass M, because of his undeniable revolution around the secondary mass m', he doesn't see, in a certain instant, this last one in the direction M-m' (Fig. 14), that forms the angle Φ with the abscissas axle of the system anchored to M, but it sees it in the direction Me -m', that forms the angle β with the abscissa of the reference system set in M' (Fig. 20) and that is

$$\beta = \phi + \alpha \, . \tag{1.47}$$

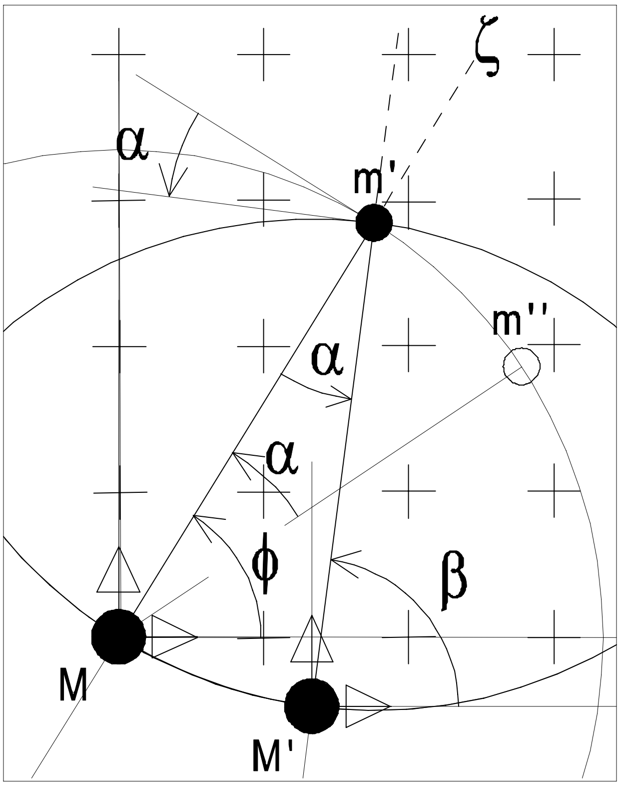

Fig. 20

Then it can be said that when it is possible to pose the relationship m/M it is rigorously equal to zero, the orbit crossed around the Sun is that one suitable in fig. 11 with the number 1, vice versa it is that one suitable with the number 2.

In addiction, and this must be underlined, only when the secondary mass is really negligible in comparison to the central one there is a biunique correspondence among the traditional reference system of the theoretical physics entirely anchored to the central mass, and that we will call local, and that one adopted in astronomy. When instead the peripheral mass is not negligible anymore then this fundamental, unconscious and implied identity comes less inexorably.

For what we will even deduce about the **sidereal time**, it is opportune to do again the calculations that have let us to reach the (1.46) in some other way. Now let's consider the formula that allows us to finish the period of revolution of m around M (in the local system). It is

$$T = \sqrt{\frac{4\,\pi^2 d^3}{G\,M\left(1 + \dfrac{m}{M}\right)}} \, . \tag{1.48}$$

Let’s calculate the variation that T suffers in function of the secondary mass m. In conclusion it is had

$$\Delta T = -\frac{1}{2}\frac{m}{M+m}\sqrt{\frac{4\pi^2 d^3}{GM\left(1+\frac{m}{M}\right)}} = -\frac{1}{2}\frac{m}{M+m}T\ . \qquad (1.49)$$

Therefore the mass m' will employ, in comparison to that negligible one m '', a smaller time to come in m', equal to

$$\left|\Delta T\right| = \frac{1}{2}\frac{m}{M+m}T\ . \qquad (1.50)$$

In this time, possessing a speed equal to

$$V_{Mm} = \sqrt{\frac{G\,M}{d}\left(1+\frac{m}{M}\right)} \qquad (1.51)$$

it will cross, in comparison to the mass m', still an arch of length

$$l = \frac{1}{2}\frac{m}{M+m}\sqrt{\frac{4\pi^2 d^3}{GM\left(1+\frac{m}{M}\right)}}\sqrt{\frac{GM}{d}\left(1+\frac{m}{M}\right)} = \pi\frac{m}{M+m}d \qquad (1.52)$$

from what we have once again that

$$\alpha = \pi\frac{m}{M+m}. \qquad (1.53)$$

But the weary of time given by the (1.53) should be instead attributed to that one employed by the central mass M to cross its partial revolution around m, only apparently employed by m '' to describe the angle α.

3 – The sidereal time

Even the period of sidereal revolution or time of revolution of a planet, fundamental greatness for the astronomy, it comes to be modified. Keeping in mind the fig. 14, we have what follows. We suppose that, to the time t=0, the central mass occupies the position M and that peripheral one the position m' of the said figure. In this instant these two masses intercept on the celestial sky the star ζ. Spent the time T, the observer anchored to M, thinking to be immovable in comparison to the fixed stars, it won't intercept ζ anymore but a star that is more ahead than this last one of an angle equal to α. Instead if he will notice the time during which the mass m realigns with ζ he will measure, by Newton, only the time of revolution that employs the mass m to turn around M (fig. 7). Therefore it will attribute to the time of revolution a smaller value than the time of total revolution.

It's true that, independently on the observer's intentions anchored to M, however the central mass will stir so that to intercept the star ζ, but if the central mass is the Sun, this last one will also be endowed with other movements that the other planets impose it and above all Jupiter. If on the other hand, in the Problem of the n Bodies, the deformations are determined suffered by the various keplerian orbits, always gotten thinking about the Sun to be fixed, the results can be very debatable (take a look at Mercury's perihelia and what we will say above all about lunisolar the precession).

We have just mentioned this matter on which we will return in a short while, we say that if, in base to the (1.46) or (1.53), we calculate the moves of the Sun provoked by every single planet we have the Chart (1.54), that must be compared with the Chart (1.32).

Pianeta	$\frac{m}{M+m}$	*Periodo anni*	*Arco solare in cento anni*	T_{SP} *Rivoluzione solare di* 360° *Anni platonici*
Mercurio	1.65×10^{-7}	0.24	44.42	2 917 604
Venere	2.45×10^{-6}	0.62	257.97	502 384
Terra	3.00×10^{-6}	1.00	194.69	665 674
Marte	3.24×10^{-7}	1.88	11.17	11 602 507
Giove	9.53×10^{-4}	11.86	5 206.33	24 893
Saturno	2.85×10^{-4}	29.46	627.63	206 491
Urano	4.36×10^{-5}	84.01	33.62	3 854 848
Nettuno	5.29×10^{-5}	164.8	20.80	6 230 769
Plutone	4.97×10^{-9}	247.7	0.00	∞

(1.54)

In this chart it has been underlined the apparent advancement of Mercury's perihelia that would also find a physics justification for it, given to a Real demotion of the Sun provoked by the mass of the planet. In effects, according to this version, it is the planet Mercury that pushes the Sun toward the point γ of 44'' and it's not the advancement of the perihelia. In addiction, a very important thing, but referable to the same identical effect, they has been underlined, the demotion of the Sun caused by Jupiter and by the Earth, the moves of the

Sun toward the same point provoked by these two planets. If we make a subtraction between these two last values it is found

$$5206.33 - 194.69 = 5011''/\sec olo \Leftrightarrow 50.11''/\,anno \tag{1.55}$$

quantity that is incredibly and practically coincident with the experimental value (1900) of 50.25 '' a year of the lunisolar precession! By this, the platonic year would be given

$$anno\ platonico = \frac{360° \times 3600}{50.11} = 25\,863\,anni \tag{1.56}$$

against the value of

$$anno\ platonico = \frac{360° \times 3600}{50.25} = 25\,791\,anni \tag{1.57}$$

before facing the problem of the precession let's re-examine once more the fundamental problem of the two bodies because it will also allow us to deepen what just mentioned on the sidereal or revolution time of the peripheral mass around the central one and that's because these two matters are intimately connected between them.

4 – On a new solution of the Problem of the Bodies

Let's analyze with more accuracy this fundamental problem that is at the base of the today's theory of the planetary perturbations, taking back the solution given by Newton, now undisputed and unanimously approved yet.

He, as already mentioned, starts from the presupposition that the two masses M and m orbit around the barycentre K and they describe the two circumferences traced in the following Fig. 21, in a more marked way, obviously an indisputable thing, if it is admitted, for the moment, the instantaneousness of the gravitational action.

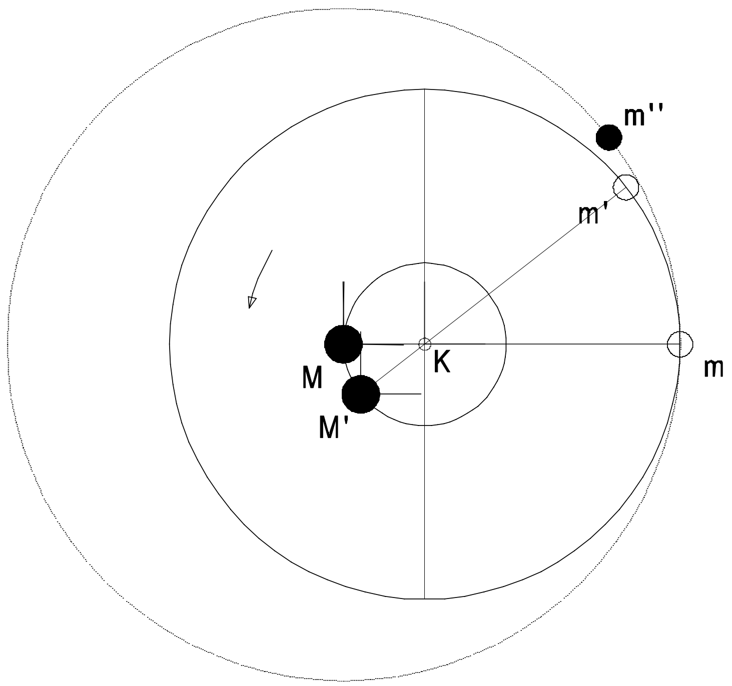

Fig. 21

Then, in comparison to the inertial system anchored to K, the mass M will describe the orbit of ray M-K and that one m of ray K-m. In more it is admitted, always by Newton, that, in comparison to the local system anchored to the central mass M, the secondary mass m won't describe the orbit of arc m-m' and of ray K-m anymore, on the contrary the greatest orbit (fig. 21), of arch m-m'' and of ray M-m around the mass M, this last is thought absolutely fixed. It's easy to see that the length of this last circumference is exactly equal to the sum of the lengths of the two circumferences described by the two masses around K. And it is the first time that, passing from a reference system to another, it changes the length of a path performed by a body, length that instead should be an absolutely and obviously invariable. This would be equal to say, moving this fact to geometry, that the surfaces and the volumes of the solids change according to the reference system.

Obviously we limit to say, for matters of brevity, that what we observed here in rather elementary way can also be underlined when it is used the rigorous analytical language that is adopted in all the essays of Celestial Mechanics in the moment in which are written the differential equations of the motion of the two bodies. About it, for the researcher of Celestial Mechanics, it will be easy to individualize when, with the said mathematical formalism, we pass from the fixed reference system (and inertial one), external to the two masses (and in that case it shows, in an unexceptionable way, the fundamental Principle according to which the barycentre K is or even it fixed or it is endowed with rectilinear and uniform motion ([4])), at the moment in which we place in the reference system anchored to the central mass M and it is implicitly and unconsciously formulated that the orbit in such case is different etc. etc.. On the subject all the innumerable essays of Celestial Mechanics can be consulted. The Italian Reader is able, for instance, to consult Zagar's astronomer essay [6, page 311 and the following ones].About it, we will be more precise in the **§ 8**.

It's, against this, evident that even in the reference system anchored to M the path described by the mass m will always however and lonely be the circumference of ray K-m and arch m-m' and not that one pointed out by Newton and accepted by the actual Celestial Mechanics!

Besides, in the passage from the system K to that one M, is thought completely **null** the secondary mass m and it is considered the central mass not equal to M anymore but it is set equal to M+m.

By this simple newtonian artifice ([5]) we find again forehead to a secondary mass m completely negligible and so therefore it cannot move that central one anymore, while this last one becomes greater and equal to M+m. By this unjustifibly the barycentre of these two masses is forced to coincide with the centre of the Sun to the lonely purpose to revert in the first problem faced by Newton in the **Principia**, that consists in the study of the possible orbits described by a corpuscle around an *absolutely fixed* centre according to the known law of the square inverse of the distances. And in this lonely and specific case He, magistrally shows, that from the hypothesis of a gravitational strength inversely proportional to the square of the distance the conic sections come down and, particularly, the experimental ellipses of Keplero. And it is just the use of this artifice the motive for which also the non inertial observer anchored to the mass M finds again the law of the inverse square of the distance.

4 This Principia is so important that from it, it's immediately reliable the famous formula $E=mC^2$ and this independently on the Narrow Relativity. Let's try to imitate the demonstration that on the subject Einstein made. Because the electromagnetic radiation manages on an any surface a pressure, an experimenter inside an isolated system, preparing of a bright source and of a reflecting surface, he could, exploiting this pressure, to continually accelerate the said laboratory, from the inside (we know instead that if we move inside a boat, we don't succeed in doing this). In fact, in the moment in which the radiation strikes a wall of the laboratory, only by practicing a pressure, it would accelerate, in base to the II° Principia of Dynamics, the said laboratory continually, violating both the Principia according to which the barycentre of an isolated system is firm or it is endowed with rectilinear and uniform motion and this in base to the I° Principio of Dynamics and also violating the Principia of the energy conservation. So that therefore all of these Principia are rightly respected even in this specific case, it is necessary to recognize that the electromagnetic energy, it has to practice an undeniable pressure, and further an inertial mass, so that, when it bounces on the reflecting surface and it strikes the other wall it succeeds in decelerating the laboratory at the same way of the boat quoted first and therefore, averagly, the barycentre of that laboratory respects the said principias. So the pression is given by

$$p = \frac{F}{A} = \frac{E}{V}$$

or rather it is given by a strength on the unity of surface or it is given by an energy on the unity of volume. From this relationship it would be drawn that

$$E = \frac{F}{A} V = F\ s = m\,a\,s = mv^2 = mC^2.$$

This demonstration of ours, that has the strong taste of a simple dimensional verification of the energy with which it is not therefore possible to determine the numerical coefficients of a formula, it is referable, as already said, to a famous reasoning done on the subject by Einstein, to show that the said relationship was also independently gotten by the theory of the narrow relativity. We reach the same result instead if we simply remember that the total electromagnetic energy not polarized is exactly the double one of the classical kinetic or inertial energy.

5 What it literally breaks the state of quiet or rectilinear and uniform motion Principia of the real barycentre K.

All of this involves the analytical results, that, as so many times underlined, consider the Sun absolutely fixed, they don't coincide with the experimental reality anchored to the reference system of the fixed stars anymore because the Sun, in this crucial reference system, it can never be considered this way because the planetary masses, even though modest, will never be null. It is born therefore the problem to adapt the theoretical results, gotten with the said tacit and unconscious free hypothesis to have to translate these last one in a reality that is different from the said implicit and tacit postulation.

In conclusion, it cannot be admitted, by the way, that the Sun stirs around the barycentre of the whole solar system and then, in the analytical developments, to deny this reality. The example of the circular orbit crossed by two masses, the one negligible and the other not, in base to which we have deduced the moves of the solar mass brought in the Chart (1.54), it reports the neckline and the heavy contradiction existing among the consequential theoretical results from certain types of reference that can be defined *local* with the global reference system constituted by the fixed stars. Divergence of results that doesn't exist, as already said, only when the orbiting mass is negligible in comparison to the central one. And therefore it could be inflicted that the law of the square distance inverse is all right only when m/M=0 can be set, hypothesis instead held truthful always from the astronomers in the large and different majority of the cases [2].

If instead we recognize that also in the system anchored to M, the mass m will keep on describing its Real orbit m-m', then the problem rises to establish with what gravitational law the said mass will describe the real orbit in comparison to the system anchored to M, once again, considered fixed. Subsequently to this phase, still transitory, then these results become essential to be traslated in the reference anchored to the fixed stars, only and irreplaceable scenery with which then there's the need, in conclusion, to make the comparisons.

To face this problem and one reasonable solution of its, let's start by imposing that, also in the system anchored to M, the mass m will owe to obviously describe its Real orbit. If in this case, we admit besides and only momentarily, by Newton, that the system anchored to the Sun is still fixed in M (Fig. 21), we immediately see that the law of strength to which the mass m comes to be subject it's not inversely proportional to the square of the distance anymore but it is of the type

$$F = \frac{Cost}{d^x}, \tag{1.58}$$

where the exponent x depends on the distance of M from K., for example, when M is on the circumference of ray K-m=d, then it is notoriously had that [see **Principia** or [5]]

$$F = \frac{Cost}{d^5}. \tag{1.59}$$

so it's evident, because M instead is at the distance

$$\overline{KM} = \frac{m}{M+m} d \tag{1.60}$$

from K, that the exponent of the (1.58) note will depend on this last and won't be never equal to two. Everything this is still obviously a consequence of the provisional hypothesis that the Sun is fixed in M.
Instead we know that the central mass M stirs on the circumference of ray K-M.

About it it's only possible to think that the theoretical orbit ([6]) by Newton (that is nothing else than the circumference of ray M-m=d), to be conform to the fundamental reference system constituted by the fixed stars and therefore to continually coincide with the real orbit m-m', it can roll on this last one, as brought in Fig. 22, even if this is, as we will see, a more precise solution, but not completely rigorous.

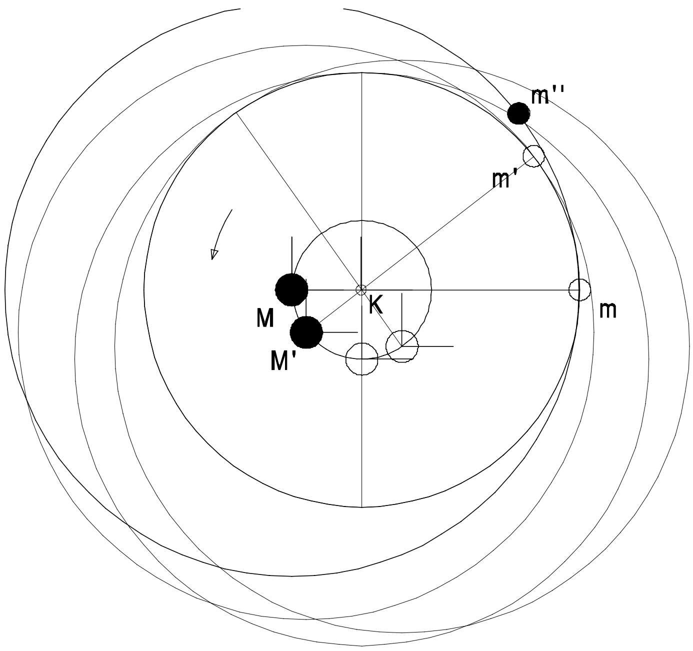

Fig. 22

By this Newton's rolling operation of the orbit on that real one, the instant centre of the Newton's orbit will describe, not really correctly, also the movement of the central mass in comparison to the system anchored to the fixed stars and therefore, even if not so precise, this time will be respected the Principia of quiet or the rectilinear and uniform motion of the barycentre K.

But it is easy to notice that if we proceed with this operation the negligible mass m'' of Newton won't never exactly coincide, instant to instant and so in the interval T, with the real mass m' because this last one has a slightly smaller speed than the first one.

In fact, the speed of m', in comparison to K, is given

$$V_{Km} = \sqrt{\frac{G\,M}{d\left(1+\frac{m}{M}\right)}} \simeq \sqrt{\frac{G\,M}{d}\left(1-\frac{m}{M}\right)} \simeq \sqrt{\frac{G\,M}{d}}\left(1-\frac{1}{2}\frac{m}{M}\right)+..+ \qquad (1.61)$$

and so

[6] That it's drawn in the local reference system M.

$$V_{Km} \simeq \sqrt{\frac{G\,M}{d}} - \frac{1}{2}\frac{m}{M}\sqrt{\frac{G\,M}{d}} + .. + . \qquad (1.62)$$

instead the speed of m in comparison to the fixed system anchored to M is given by

$$V_{Mm} = \sqrt{\frac{G\,M}{d}\left(1+\frac{m}{M}\right)} \simeq \sqrt{\frac{G\,M}{d}} + \frac{1}{2}\frac{m}{M}\sqrt{\frac{G\,M}{d}} + .. + . \qquad (1.63)$$

by the comparison of the two last formulas is seen that the speed of the secondary fictitious mass of Newton (1.63) is greater than the speed of the real mass ([7]) (1.62) and therefore if the orbit of Newton continually rolls on the real orbit it is never had that the point of contact between the two circumferences coincides, instant to instant, with the position that m assumes on the real orbit and in addiction, after the period of time equal to T ([8]), the secondary mass of Newton won't occupy the common position of departure of the two masses anymore but it will slightly be more ahead.

In fact, the difference of speed between the (1.63) and the (1.62) is equal to

$$\Delta v_m \simeq +\frac{m}{M}\sqrt{\frac{GM}{d}} + .. + . \qquad (1.64)$$

By this more speed in the fictitious mass, after a rolling period equal to T

$$T = \sqrt{\frac{4\pi^2 d^3}{GM\left(1+\frac{m}{M}\right)}} \simeq \sqrt{\frac{4\pi^2 d^3}{GM}\left(1-\frac{m}{M}\right)} \simeq \sqrt{\frac{4\pi^2 d^3}{GM}} - \frac{1}{2}\frac{m}{M}\sqrt{\frac{4\pi^2 d^3}{GM}} + .. + \qquad (1.65)$$

it will have crossed an arch

$$\Delta l \simeq +\frac{m}{M}\sqrt{\frac{GM}{d}}\left(\sqrt{\frac{4\pi^2 d^3}{GM}} - \frac{1}{2}\frac{m}{M}\sqrt{\frac{4\pi^2 d^3}{GM}}\right) + .. + \qquad (1.66)$$

or

$$\Delta l \simeq 2\pi\frac{m}{M}d - \pi\left(\frac{m}{M}\right)^2 d + .. + \qquad (1.67)$$

since the relationship m/M is always very small then it's strongly right the square of this quantity and therefore, in conclusion, it will be had that the advance angle of the fictitious mass m'' in comparison to that real one m will be equal to

$$\alpha^{rad} \simeq 2\pi\frac{m}{M} \equiv \alpha° = 360^{\circ}\frac{m}{M} + .. + \qquad (1.68)$$

---

[7] That one the mass m has got in comparison to the barycentre K or to the reference system of the fixed stars.

[8] In the hypothesis the time of revolution is the one noticed by the observer anchored to K, a very rigorous thing to be said but impossible in realizing.

a value that is practically coincident with the angle of reciprocal revolution of the central mass already found by the simple considerations of geometric character (see the formula (1.4)) and that is

$$\alpha^{rad} = 2\pi \frac{m}{M+m} \simeq 2\pi \frac{m}{M} \equiv \alpha^{\circ} = 360^{\circ} \frac{m}{M}. \quad (1.69)$$

Therefore, if we roll the orbit of Newton, without any skid, on the real orbit, not only we have that there will be no instant coincidence between the point of contact of the two circumferences but we also get that, after a period T, the point of contact between the two circumferences will never coincide with the point of departure m (Fig. 11), but it will be more ahead than this last one of the angle given by the (1.53). this last fact, that could appear an insurmountable problem is, as we immediately will see, only a forgery enigma.

To realize this last affirmation we have to abandon, for a moment, the results we have had with the theory of Newton and to return to simple considerations of geometric character. Let’s suppose that to the time t=0 the two masses M and m occupy the suitable position in Fig. 23. It's easy to see that we can also roll the inside circumference on the external one,

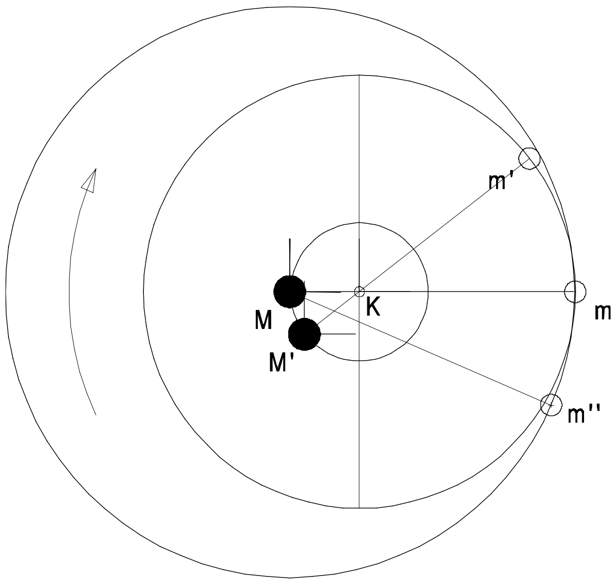

Fig 23

as suitable in the sketch, and then the varying point of contact between them will coincide with the instant position that assumes m' on the circumference of ray K-m.

It's however entirely evident that, passed the time T of revolution measured by K (remembering that the mass m, in comparison to K, in the time T, it crosses the whole circumference of ray K-m), the point of contact between the two circumferences won't take back the departure position m anymore on the contrary the position m''. <u>it occurs an undeniable and non-eliminable retrogradation.</u>

In fact it is simple to verify that, stretching the length of the circumference of ray K-m on that greater one of ray M-m, we will go individualizing the m'' such as that m''-m has a length equal to

$$m - m'' = 2\pi d - 2\pi \frac{M}{M+m} d = 2\pi d \frac{m}{M+m} \qquad (1.70)$$

and so the angle m’’Mm is equal to

$$m''M\ m = \alpha = 2\pi \frac{m}{M+m} \equiv 360° \frac{m}{M+m}. \qquad (1.71)$$

in consequence of this, even the times of revolution measured from the two observatories will be different. In fact the complete revolution of the secondary mass around the principal one, for K, it will be determined in the moment in which the point of contact of the said circumferences will reassume the position of departure m while, for the observer M, this final instant will be determined when the said point of contact will be in m''.

Vice versa, if we stretch the whole length of the greatest circumference on the smaller one we will have that the point m'' will have gone beyond the initial point m of an angle always given by the (1.71) and therefore there is an advancement. If, in this last case, we define as time of revolution T that one measured by the inertial observer K, the non inertial observer M, that notices an advancement of the point of contact, it will find that this last one will slightly coincide with the point of departure m only after a time slightly inferior to the one noticed by K and so that the subtraction is equal to

$$\Delta T = \frac{m}{M} T. \qquad (1.72)$$

in fact, we have already seen that, in the time T, the mass m revolutionizes around M of an angle

$$\alpha = 360° \frac{M}{M+m} \qquad (1.73)$$

then we will have that the time to describe the angle given by the (1.71) is deducible from the simple proportion

$$T : 360° \frac{M}{M+m} = \Delta T : 360° \frac{m}{M+m}, \qquad (1.74)$$

from which comes down the (1.72). Therefore since in the system M the mass m has an angular speed equal to

$$\phi_{Mm} = \frac{360^o}{T} \frac{M}{M+m}, \qquad (1.75)$$

multiplying this last for the said time ΔT, we obtain the angle given by the (1.71).

The problem just exposed, if obviously it is not exempt from errors, it would ask for a substantial afterthought on these hinge elements of the Celestial Mechanics.

As it is seen by this simple analysis, the two reference systems K and M, are between them completely incommensurable in the sense they will never give the same coincident results except for the case, very banal, in which the mass m is completely negligible in comparison to the central one. If priority is given to the inertial reference system K and therefore the external circumference rolls on the internal one, the observer loyal to M finds an advancement of the orbit described by the secondary mass. Vice versa, a retrogradation is found. This however it is not at all an insoluble ambiguity of the physics because it has never pretended the laws regulating the universe to be the same ones between two reference systems the one inertial K and the other, not inertial M, *thing that instead happens with the known procedure of the Celestial Mechanics.*

To find the physical law in the case in which priority is given to the one or to the other of the two observatories is still the insuperable Newton coming to help us, and the matter is really crucial. In fact He, in his everlasting work, among the so many problems he resolves, he faces a *meaningfully* analogous problem.

Newton knew very well that the perihelia of the planets (and above all the lunar perigeal (take a look at the ([9])))they sensitively advance in the sense of revolution of the planet itself. In effects, the planetary orbits not always have the most greater axle fixed in the space, as so it would like it to be the square distance inverse law, but this axle rotates, *with varying angular speed*, in the sense of revolution of the planet itself, describing the rose orbit of Fig. 24.

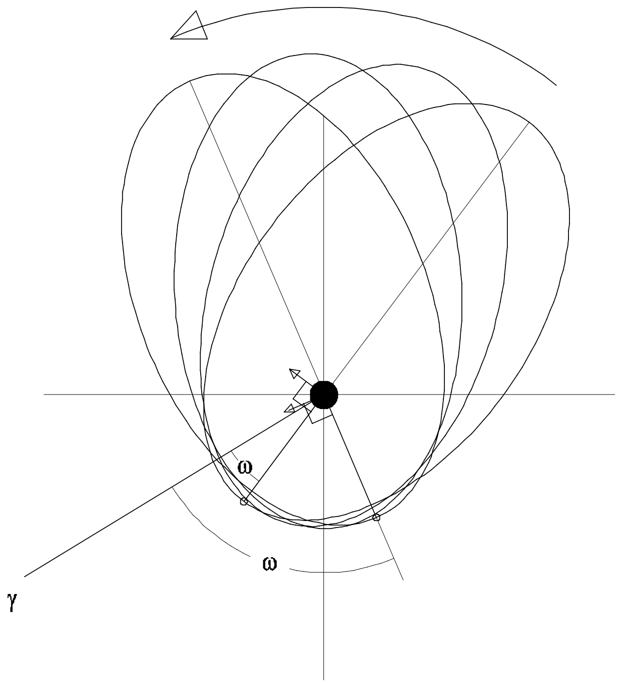

Fig. 24

Newton wondered then what gravitational law can describe this type of orbits. In his work we find, with a really inimitable analytical deduction, that the exponent of the relationship

$$F = \frac{Cost}{d^x} \tag{1.76}$$

it is given by the following formula

$$x = 3 - \left(\frac{360°}{360° + \alpha^o} \right)^2 \equiv 3 - \left(\frac{2\pi}{2\pi + \alpha^{rad}} \right)^2, \tag{1.77}$$

[9] We have already seen that for the binary Earth-moon system the relationship m/M is the maximum one that is possible to verify in the solar system

where α is the advancement or the retrogradation of the orbit on a complete period of revolution T. From the (1.77) is deduced that if α=0, the exponent of the (1.76) is exactly equal to 2. If there is an advancement it is greater than 2 while, in the case of a demotion, a smaller exponent of 2 will be had. It is, on the subject, the case to remember that Newton realized whether to be able to give account of the motion of the lunar perigeal (see Fig. 24), that reports the maximum move ahead of the whole solar system, he would have had to write the formula

$$F = G\frac{M\ m}{d^{2.01646}} \tag{1.78}$$

and he thought that this showy discrepancy could be all brought back to the action of the Sun on the lunar orbit. Since it didn't have other available effects to which attribute a concause, he opted for an exponent of **d** exactly equal to 2, but he never succeeded in showing that the difference equal to 0.01646 would entirely be attributable to the action of the Sun. Rather Newton succeeded in attributing the half of this discrepancy to the Sun only, but he didn't make scruple to double his theoretical result, clearly making up the results. This is not a blamable behavior from a so important Teacher; He perhaps thought that others would have resolved the one that cannot certainly be called a marginal problem, that resurfaces, after so much time, just about the advancement of Mercury's perihelia.

This is, in our notice, the most important and *meaningful* point of the Principia that worried, up to his death, both Newton and Roger Cotes the publisher of the II° edition of this masterpiece, who held to clearly specify , in its preface, and regarding this fundamental matter, that in the **Principia** was shown this sensitive deviation was all attributable to the action of the Sun only. Even what, very later, Newton devoted to this problem he didn't succeed in resolving it. And if he has not succeeded in it, is certainly caused by the fact the cause of this deficiency perhaps finds its explanation in the neglected role that the orbiting masses have on the subject, as we are now seeing.

At the half of '700 this unresolved problematic explodes again. Resoundingly the French mathematician Clairaut (1747) affirms that the gravitational theory of Newton doesn't succeed in explaining (already known thing) the whole *advancement* of the lunar perigeal (let's observe, as already mentioned, there is an undeniable and evident simile between this ancient problem list and the one related to the so many *inexplicable* advancement of Mercury's perihelia) and it hypothesizes in worth the existence of a corrective strength of the gravitational law.

Then, as many resoundingly, Clairaut announces to have succeeded in resolving this problem list (overcoming the magistral mathematical ability of the insuperable Newton?) bringing all back to the known gravitational law. We will face the study of this note by Clairaut in a following work, but we immediately say that this problem list is not exquisitely referable to the difficulty of mathematical character of the not yet resolved solution of the Problem of the Three Bodies, on the contrary exactly to the present criticisms turned at the base of the C.M. and that is to the ancient, physical and not mathematical Problem of the two Bodies.

After mentioned this, if we apply a spin or an advancement or rolling to the orbit of Newton of an angle equal to

$$\alpha = \pi \frac{m}{M+m} \tag{1.79}$$

we have, from the (1.77), that the exponent of the (1.76) is equal to

$$x = 3 - \left(\frac{1}{1 + \frac{1}{2}\frac{m}{M+m}} \right)^2 \simeq 3 - \left(1 - \frac{1}{2}\frac{m}{M+m} \right)^2 \simeq 2 + \frac{m}{M+m} \tag{1.80}$$

and so the law, <u>in the fixed reference system</u> <u>M</u>, would be given by the ([10]) a relation of the type

$$F \simeq G\frac{M\ m}{d^{2+\frac{m}{M+m}}} \simeq G\frac{M\ m}{d^{2+\frac{m}{M}}} \tag{1.81}$$

which produces an <u>apparent</u> advancement of the every secondary mass perihelia given to the <u>real</u> recoil of the Sun in conformity what reported in the chart (1.54). In the case of the lunar perigeal it becomes

$$F \simeq G\frac{M\ m}{d^{2+\frac{1}{81.3}}} = G\frac{M\ m}{d^{2,012300}} \tag{1.82}$$

and it leaves to the action of the Sun and all the other planets only 0.00416.

Instead in the case in which is wanted not only to impose the said rolling but that there would also be the instant coincidence among the point of contact between the two circumferences and the position that the mass m assumes on its real orbit then the said advancement (or retrogradation) doubles (Formulas (1.4) and (1.5)). If, in this last case, the priority is given to the observer K then the orbit in comparison to M goes to roll on that smaller one and it is had an advancement. In such case it is had

$$F \simeq G\frac{M\ m}{d^{2\left(1+\frac{m}{M+m}\right)}} \simeq G\frac{M\ m}{d^{2\left(1+\frac{m}{M}\right)}}, \tag{1.83}$$

vice versa it is had

$$F \simeq G\frac{M\ m}{d^{2\left(1-\frac{m}{M+m}\right)}} \simeq G\frac{M\ m}{d^{2\left(1-\frac{m}{M}\right)}}. \tag{1.84}$$

[10] The (1.81) coincides, in the case of Mercury, with the one proposed by the astronomer Hall to resolve the problem of Mercury's perihelia. In fact He proposed to assume, without any justification, as exponent of d the fixed number 2.00000016. And if the relationship between the mass of Mercury and the one of the Sun is made, it is had that m/M=0.000000165.

Obviously the (1.83), in the case of Mercury would give the double of the inexplicable advancement of the perihelia and, in the case of the Moon a greater advancement of the one experimentally observed.

And it's to be noticed that Newton's law, in the system anchored to M

$$F = G\frac{M\ m}{d^2}\left(1+\frac{m}{M}\right) \tag{1.85}$$

it behaves that, on the terrestrial surface, the acceleration is given by the relationship

$$g = G\frac{M}{d^2}\left(1+\frac{m}{M}\right) = g_o\left(1+\frac{m}{M}\right) \tag{1.86}$$

and therefore the ways the serious earthlings fall also depend on the falling masses. By Newton, a heavier body will slightly employ a fall time slightly smaller than the lighter one.

In conclusion the problem of the two bodies, in the reference system anchored to the mass M, finds, thanks to Newton, various solutions among the most opportune one can be chosen. But this choice also requires from the reflections on the most correct definition of the sidereal time and therefore we will dedicate a following note to this. Momentarily, for the study of the precession phenomenon, we will hold valid the moves of the Sun deduced by the theory of Newton leaving the sidereal times unchanged.

Regarding the preceding relationships we limit, for the moment, to mention what it follows. The relationships (1.81), (1.83) and (1.84) behave a commune thematic but they have opposite physical effects. But before this sign perhaps the following fundamental premise is opportune. All the physical laws today we know have an only and indisputable matrix: they come down from experimental facts, there no need to make illusions about it.

Not by chance Newton deduces the *universal* gravitation from the experimental laws of Keplero. This involves that the square of the inverse distance law derives from observations done in a limited window of the universe and in addiction by neglecting the role of the secondary masses. The cosmological observations of these days put in crisis all of the actual gravitational theories (for example, dark matter etc.). On the other hand the invalidity of Newton and Coulomb's formula for very small distances is shown by the old and **innate singleness** ([11]) these relationships have when d=0: it is not possible that two masses (or charges), for d tending to zero, in spite of the modesty of their masses or charges, they would instead practice among them a strength extending to the infinite, and this fundamental singleness has never been removed.

We are legitimately able therefore to suspect that these laws are next to the reality in a certain spatial interval only and that therefore they can differently vary, both when d extends to the infinite and when it extends to zero, by well different laws from the inverse distance square. If then, suppressing some actual, and for now, impossible directed

[11] **This singleness is really interesting.** We have already seen that the ultraviolet catastrophe can have another solution (www.carlosantagata.it), completely classical, without resorting to the unproved discontinuity sprung by the hypothesis of Planck. In fact the actual quantum Mechanics still suffers from singleness (see nobel Abdus Salam).

experiences, the removal of certain clear contradictions already suggests some theoretical changes, they should be taken with a lot of favor, thing that is well difficult to verify in the actual world of the homo sapiens.

After said this, regarding the preceding relationships there is to say what it follows. First of all they are not invariant in comparison to the length unity used. This would suggest the choice of a fundamental unity length, removing the actual total arbitrariness. It is longly thought that some greatnesses of the physics, such as the constant of fine structure and the classical ray of the electron, could acquit to this assignment, but not yet we know whether to introduce them to come to an unification of the interactions nowadays known, unification that, to the state of the science, it only appears as a distant and unattainable mirage.

If the (1.81) were valid it would fall first the lightest body and then the heavier one([12]) and this would establish a further analogy among the gravity and the electric field. This phenomenon of the fall difference foreseen by the (1.81) curtains as soon as to grow weak thin to entirely annihilate when **d** has the tendency to coincide with a 1 cm. [c.g.s.]. Under this length the inverse phenomenon would begin. It would begin to fall first the heaviest body.

In addiction, as soon as the secondary mass increases the orbit would extend up to become spiral. This could be in tuning with the expansion of the universe and with the palpable fact our solar system is characterized by a strong difference between the central mass and those peripheral ones (even the atom has this characteristic). Not by chance the great planetary masses are toward the outside of our solar system and therefore there could be in action a slow leaving of them. Also the Moon, according to the last results of the Nasa, reports the tendency to estrange from the Earth as the time passes.

In the case of the identical masses, the (1.81) can be written

$$F = \frac{G}{d}\frac{M\ M}{d^2} = G'\frac{M\ M}{d^2} \qquad (1.87)$$

from (1.87) is also had

$$G' = \frac{G}{d}. \qquad (1.88)$$

Then, leaving the inversely square of the proportional distance force law unchanged, we would have a gravity *constant* varying with the distance. For great distances gravity would decrease with a greater speed, otherwise it would be had for dwarfish distances. For example, let's consider the ray of Bohr,

$$R_{Bohr} = \frac{e^2}{mC^2}137^2 = R_e\,137^2, \qquad (1.89)$$

[12] In accord with the re-reading of the experiments done by Eötvös made by Fischbach and some others "**Reanalysis of the Eötvös Experiment**" Phys. Rev. Lett. 56, 3-6 (3 January 1986).

having pointed out R_e the classical ray of the electron and by 137 the inverse of the constant thin structure. From this derives that, between two unitary masses, set to the distance equal to the ray of Bohr, G' would assume the meaningful value

$$G' = \frac{G}{R_{Bohr}} = \frac{6.66\times10^{-8}}{2.82\times10^{-13}\times137.01^2} = 12.58 \cong 4\pi \quad (1.90)$$

this strange and recurrent 4π in the electric phenomenas could it perhaps constitute a first pale sign between a possible connection among gravitational position and electric position?
The Nobel Abdus Salam spoke of a super gravity. If then (1.90), to distances of the ray of Bohr, gravity became about 190 million times greater of the one measured in the macroscopic world we really would owe to agree with him. Let's imagine what would it happen to the distance of Planck.

Instead if the (1.83) were valid everything just said would be reversed.

Before leaving this fundamental matter on the divergence from the inverse distance square law, even regarding the mentioned singleness of which it suffers the formula of Newton and the one by Coulomb, let's shortly say what it follows, postponing the Reader to the website www.carlosantagata.it [*] for the close examinations of the case. In the quoted site [*] we have shown the validity of a new quantum relationship ([13])

$$\lambda = 2\pi 137\psi \quad (1.91)$$

which ties vibration amplitude of an electric charge ψ to the electromagnetic wave length associable to it λ.

The formula (of experimental genesis) of Planck's energy is

$$E = \frac{\frac{hC}{\lambda}}{\exp(\frac{hC}{kT\lambda})-1}. \quad (1.92)$$

the (1.91) can also be written [*] (e is the electron charge)

$$\frac{e^2}{\psi} = h\nu = \frac{hC}{\lambda} \quad (1.93)$$

and so the (1.92) becomes

[13] It's possible to show [*] that the known and unshown Plank's formula $E = h\nu n$ is nothing else than an **absolutely** classic and more complete resonance condition between the external forcer and the dipole. In fact energy cannot pass from a system to another if not through the **important** and uneliminable resonance phenomenon and that a formula entirely analogous to the (1.91) can also be written for the gravitational radiations [*]. Besides it is easy to see that the energy quantum $h\nu$ is exactly equal to e^2/ψ.

$$E=\frac{\frac{e^2}{\psi}}{\exp\left(\frac{e^2}{\psi kT}\right)-1}=\frac{\frac{e^2}{\psi}}{\exp\left(\frac{e^2/\psi}{m\overline{C}^2}\right)-1},\tag{1.94}$$

with $\overline{C}$ variable from case to case. So it's immediate to conclude saying that the existing average Colombian strength in an electromagnetic dipole is equal to:

$$F=-\frac{\partial E}{\partial\psi}=\frac{e^2}{\psi^2}\frac{1}{\exp\left(\frac{e^2}{\psi m\overline{C}^2}\right)-1}\left[1-\frac{\frac{e^2}{\psi}}{m\overline{C}^2}\frac{\exp\left(\frac{e^2}{\psi m\overline{C}^2}\right)}{\left[\exp\left(\frac{e^2}{\psi m\overline{C}^2}\right)-1\right]}\right]\tag{1.95}$$

so its course is represented in the Fig. 25

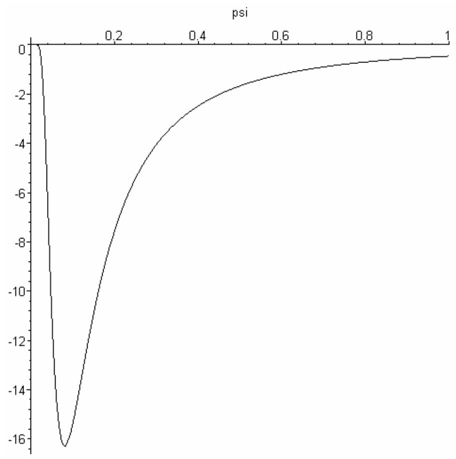

Fig. 25

from which is seen how, as soon as the distance ψ of the two charges of the dipole increases, we are nearest and nearest close to the known macroscopic formula by Coulomb (Black Body=Rayleigh-jeans) while, for the distances extending to zero, the strength becomes null and therefore said singleness is eliminated. This is probably given by the annihilation of the two charges with consequent issue of electromagnetic energy. If the job of separation or annihilation of the dipole is calculated in base to the (1.94) it is found that energy spent for this trial is equal to

$$E=m\,\overline{C}^2,\tag{1.96}$$

where $\overline{C}$ can be any speed that only in very particular cases (matter to plasma state) coincides with the speed of the light (Temperatures of the order of million of °K (see [*]). If instead the law of Coulomb had been valid we would obviously have had to spend an infinite energy! The generalization of Einstein formula (1.96) would bring to the conclusion that cold fusion phenomenas could be more real. To finish it is observed that if from the analysis of the stars motion inside a spiral galaxy the law of strength is drawn, a gravitational law with a very similar course to the one of the Fig. 25 is found. In fact the

stars found very close to the center of the galaxy have a speed that almost linearly increases (elastic law of Hooke = Iukawa) with the distance, in full contravening to Keplero's laws, which are respected for those stars toward the outskirts of the galaxy itself only.
This seems to be a further simile between macro and microcosm.

5 - A small rightful homage to the insuperable Newton

To realize that the Moon continually *falls* around the Earth for the same cause for which an apple falls on the terrestrial surface it is one of the greatest and incomparable discoveries of every time.

If indeed then the law of gravity varies with the inverse square of the distance then it is easy to verify the reliability of this hypothesis (at the times of Newton the terrestrial ray and the distance Earth-moon were not known very well, thing that instead we know).

The Moon orbits around the Earth to an average distance of cm. 0.3844 x 10^{11} and the *revolution period* is equal to 27.32166. The Earth has an equatorial ray of cm. 6378.099 x 10^5 and a polar ray of cm. 6356.634 x 10^5 . So we will assume an average ray of cm. 6367.365 x 10^5 . The acceleration of gravity on the terrestrial surface and has got average latitudes is equal to 980.665 cm/sec^2.

By these data it is possible to calculate the acceleration to which the Moon is subject on its orbit. It is given

$$a_L = \frac{v^2}{d} = \left(\frac{2\pi d}{T}\right)^2 \frac{1}{d} = \frac{4\pi^2 \times 0.3844 \times 10^{11}}{\left(27.32166 \times 24 \times 3600\right)^2} = 0.27233\left[cm/\sec^2\right]. \quad (1.97)$$

If it is true that the acceleration of a body immerged in the gravitational field is of the type

$$a_L = \frac{Cost}{d^2}, \quad (1.98)$$

in the case of the Moon we have

$$Cost = 0.27233 \times \left(0.3844 \times 10^{11}\right)^2 = 4.024039 \times 10^{20} \quad (1.99)$$

and therefore on the terrestrial surface the acceleration should be equal to

$$a_T = \frac{Cost}{R^2} = \frac{4.024039 \times 10^{20}}{\left(6367.365 \times 10^5\right)^2} = 992.527\left[cm/\sec^2\right], \quad (1.100)$$

against the 980.665 measured, with a coincidence of 98.8%. But we cannot neglect the small differences.

In effects Newton performed a more simple calculation. If the law of gravity hypothesized by Hooke is true the simple proportion will have to be verified

$$a_L : \frac{1}{d^2} = a_T : \frac{1}{R^2} \quad (1.101)$$

from which follows

$$\frac{d}{R} = \sqrt{\frac{a_T}{a_L}}. \quad (1.102)$$

by the experimental data we have

$$\sqrt{\frac{a_L}{a_T}} = \sqrt{\frac{980.665}{0.27233}} = 60.008\,. \quad (1.103)$$

Newton chose, obviously a bit ad hoc, the value of the relationship d/R equal to 60, while it is equal to

$$\frac{d}{R} = \frac{0.3844 \times 10^{11}}{6367.365 \times 10^5} = 60.37\,. \quad (1.104)$$

But, as so many times repeated, Newton has corrected the (1.101), in fact, just in the problem of the two bodies it also has to intervene the mass of the falling body. On the other hand the Moon, falling on the Earth, it would certainly employ a smaller time than the one a common grave allowed to fall on the terrestrial surface, falling from a distance equal to the one of the Moon (instead Newton kept on thinking that they would be fallen together, despite he had corrected the III° of Keplero with the introduction in it of the masses of the planets (and therefore of the falling bodies). In fact the (1.100) should be more exactly written

$$a_L : \frac{\left(1 + \frac{m_L}{M}\right)}{d^2} = a_T : \frac{\left(1 + \frac{m_g}{M}\right)}{R^2} \quad (1.105)$$

and because the mass of the grave m_g is always negligible in comparison to the Earth's one it is also had

$$a_L : \frac{\left(1 + \frac{m_L}{M}\right)}{d^2} = a_T : \frac{1}{R^2} \quad (1.106)$$

from which follows

$$\frac{a_T}{a_L} = \frac{d^2}{R^2\left(1 + \frac{m_L}{M}\right)} \quad (1.107)$$

and therefore in conclusion it is had that

$$\sqrt{\frac{a_T}{a_L}} = \frac{d}{R}\frac{1}{\sqrt{\left(1+\frac{m_L}{M}\right)}} = \frac{0.3844\times10^{11}}{6367.365\times10^{5}\sqrt{1+\frac{1}{81.306}}} = 60.002\,, \qquad (1.108)$$

and the result is not improved at all. Even if therefore Newton corrects the law of fall of a body according to the relationship

$$g = g_o\left(1+\frac{m}{M}\right), \qquad (1.109)$$

revisiting the law of Galilei, the result remains unchanged.

But if, as we have underlined, the Moon, in the time of revolution T doesn't effectuates a whole angle of 360° around the Earth because also this last one, itself, makes a small angle of revolution around the Moon, when we calculate the acceleration that the Moon suffers from the Earth we should consider that it, in the said time T, crosses an angle equal to

$$\alpha_L = 360° - \frac{1}{2}360°\frac{m}{M+m} \qquad (1.110)$$

and therefore the length of the orbit that competes it, is equal to

$$l = 2\pi d\frac{\left(1+\frac{1}{2}\frac{m}{M}\right)}{\left(1+\frac{m}{M}\right)} = 2\pi d\times0.9939246664\,. \qquad (1.111)$$

this involves that its real acceleration ([14]) will be

$$a_L = 0.2690347647 \qquad (1.112)$$

and so the relationship

$$\sqrt{\frac{980.665}{0.2690347647}} = 60.374 \qquad (1.113)$$

for which is had, this time, a 99.99% coincidence.

Instead the path the grave does falling on the terrestrial surface, even it, as a principle, entirely apparent or deceptive, is all to be attributed to the real path of the grave itself, because of the extreme littleness of its mass towards the one of the Earth. Here we find again the role of the Principia of the Absolute Relative.

[14] In fact, with Newton, we also attribute to the Moon the acceleration the Moon imposes to the Earth.

6 – The lunisolar precession

Until here we have discussed of the existing neckline among the *local* reference system used in the C.M. and in theoretical physics and the one anchored to the fixed stars and we have been able to observe as the inexplicable advancement of Mercury's perihelia and the small difference of the verification of the inverse square of the distance law, made by Newton in the case of the Moon, they can find an explanation, and this invoking the removal of the presumed and unbearable heliocentrism.

The two cases now mentioned are just a little thing if the actual explanation of the lunisolar precession is examined. In fact in this particular case the masses of all the planets enter the game and if, as already said, it is true that the planets orbit around the Sun, even this last mentioned is forced to rotate around the varying center of the masses. But even here the Mechanics Celestial starts from the presupposition that the Sun is always absolutely however fixed denying, once again, the Principia of quiet or rectilinear and uniform motion of said barycentre besides shown by the C.M.. And in this case it doesn't deal with 43 - 44" a century on the contrary of 50" a year. To be able to face this matter, for clarity of exposure, let's make some brief premises.

Let's consider the Fig. 26. In it, the ecliptic S'γG' is drawn, and it represents the apparent path the Sun effectuates on the celestial sky (fixed stars) in one year around the Earth T. In addiction, in the lateral prism, the Earth is brought, magnified, with the terrestrial equator and the plan on which the ecliptic lies for underlining the inclination of the terrestrial axle and the equatorial swelling equally tilted in comparison to the ecliptic.

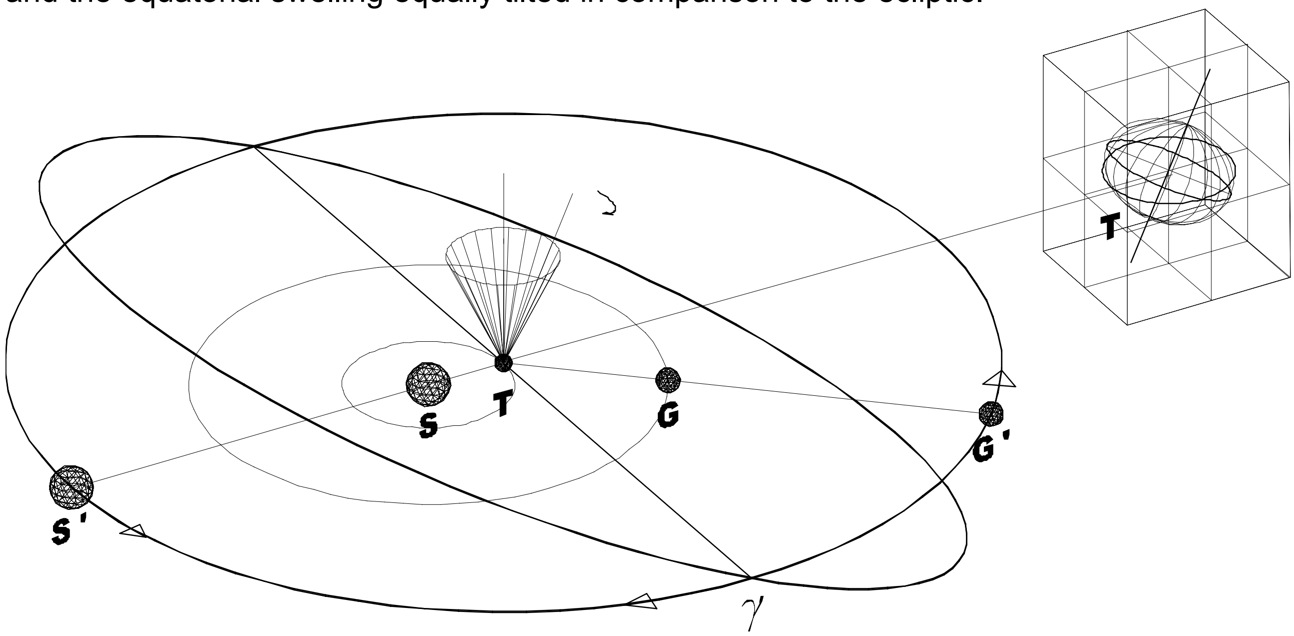

Fig. 26

By the Copernican system (1543), in effects is the Earth T to completely effectuate (or it would effectuate) around the Sun S the smallest circumference of the figure in counterclockwise sense. (Instead, in base to what in precedence established, not all of this revolution can be attributed to the generic planet. Consequently the Sun, making pivot on the Earth, curtains to go of proper motion toward the point γ, always in counterclockwise sense). After said this, obviously the terrestrial observer doesn't see the Sun in the point S but he sees it projected in S', on the celestial sky.

In the said figure it is also represented the orbit that Jupiter effectuates around the Sun, and even it is seen by the terrestrial observer projected on the celestial sky in G'. In effects the orbit of Jupiter has a small inclination on the ecliptics (1° 18') that here is neglected. In the same sketch it is also represented the celestial equator that is the terrestrial equator, also it projected on the celestial sky. Since the daily rotation axle of the Earth North-south is tilted of about 23° in comparison to the ecliptics, the celestial equator and the ecliptic intersect among them on the axle T-γ.

When the Sun, the Earth and the point γ result lined up we have the equinoxes (Fig. 26) and these are the moments in which the durations of the day and the night are identical. The semistraight T-γ, (the point γ can be considered to an infinite distance both from the Earth and from the Sun), it represents an axle of the reference system and it is of fundamental importance for the astronomy. In fact all of the parameters of an orbit of a celestial body are reported to the axle connecting the center of the Earth (geocentric system) or the center of the Sun (heliocentric system) with the said point, as it is better clarified in the following figure (Fig. 27). On how this point γ is astronomically determined there would be something to say but, for the moment, we skip this thing.

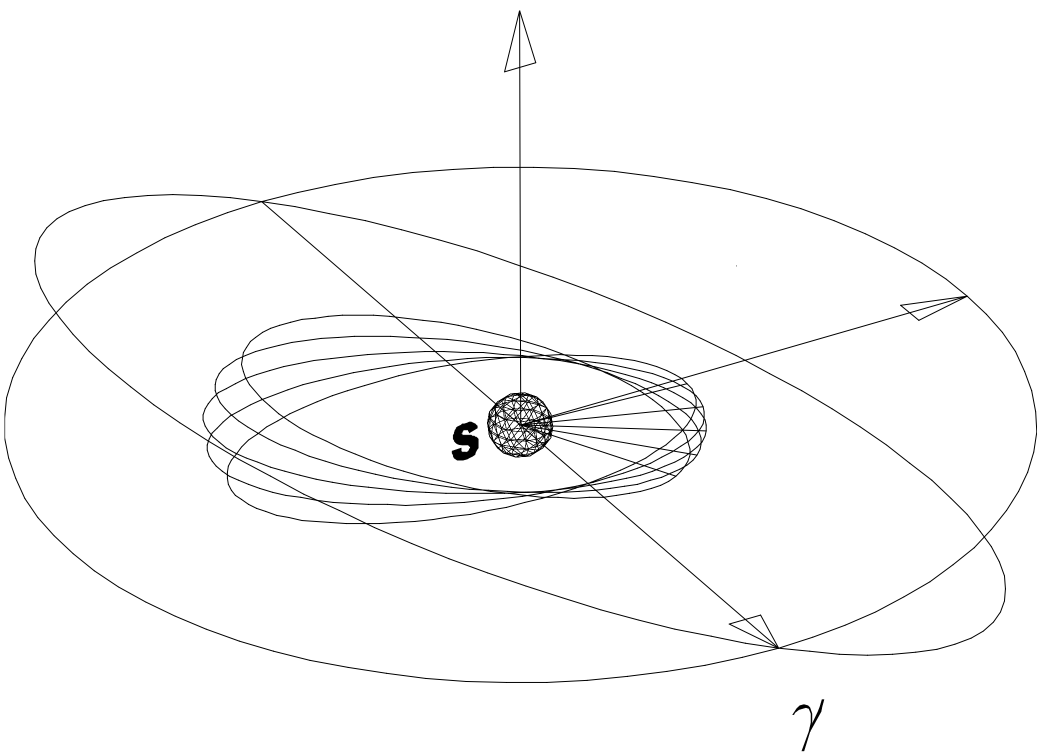

Fig. 27

While in the Fig. 26 is represented the geocentric reference system, in the Fig. 27 is represented the heliocentric reference system and the elliptic orbit of a planet whose perihelia advances in the time in comparison to the point γ, always in counterclockwise sense.

Now let’s reconsider the Fig. 26.

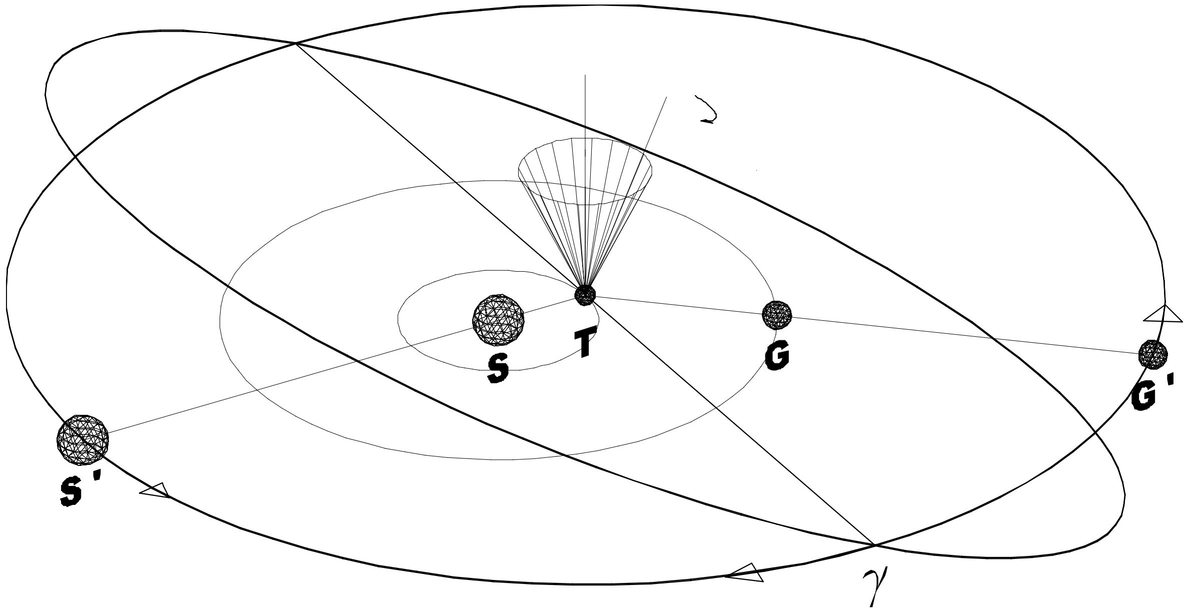

Fig. 26

If we say the Earth rotates around the Sun in a time equal to 325.25 days that's because this is the time that passes because the Sun apparently transits again for the point γ. The Fig. 28 better illustrates this thing.

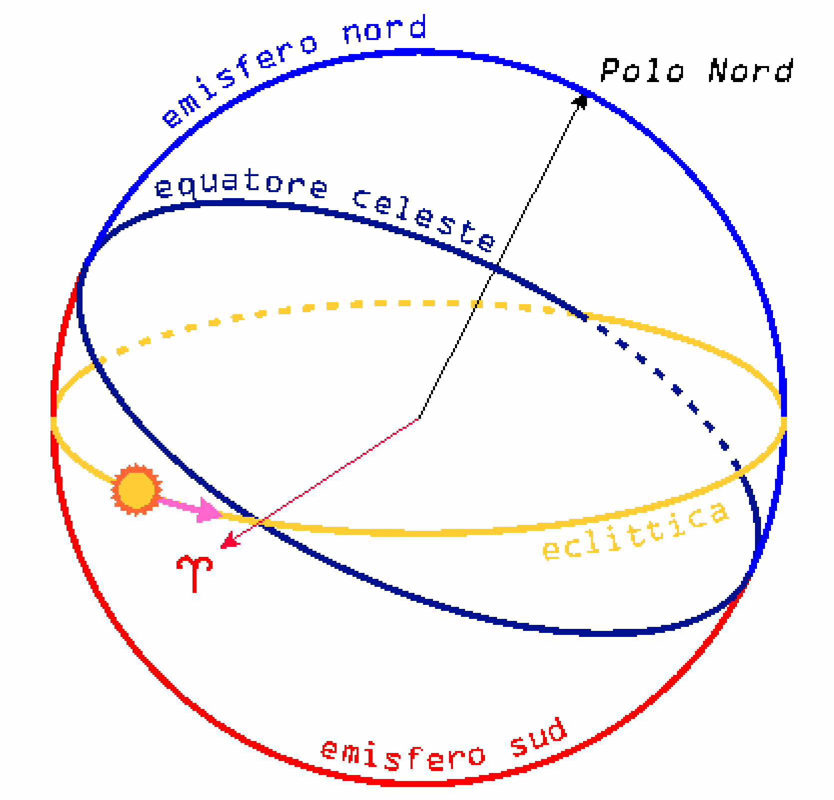

Fig. 28

As it can be seen by it, when the Sun passes for γ, it also transits from the South to the North hemisphere. That's why we see it higher in the summer and lower in winter.

But it happens that the Sun, every year, reaches the point γ ([15])with an advance of about $20^m\ 23^s$! This grandiose and evident celestial phenomenon, so fluorescent that perhaps it was already discovered in the II° century B. C. by the astronomer and mathematician Ipparco of Nicea, currently it is explained attributing the whole measured relative movement among the Sun and the point γ only to this last one and therefore still thinking the Sun <u>absolutely fixed in the space, despite the considerable recoils that it suffers above all from Jupiter</u>. The Fig. 29 illustrates the explanation of Newton.

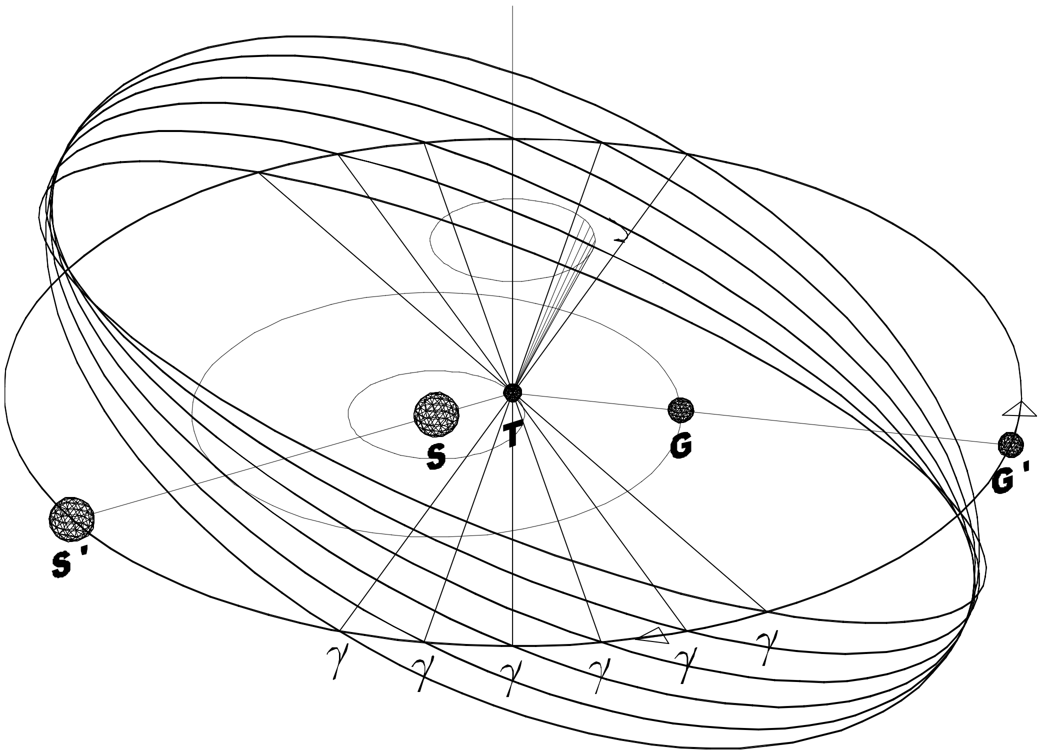

Fig. 29

According to this actual version, the celestial equator suffers a hourly skid on the ecliptics, and therefore opposite to the counterclockwise motion of the planets around the Sun, that makes that is only the point γ to go towards the Sun, if we consider, as so many times said and repeated, this last one absolutely fixed (or better endowed with the lonely apparent motion of revolution of the Earth around it).

Then this *relative* movement of well 50'' of sessagesimal arch a year, measured among the Sun and the point γ, it would be all to be written to the gyroscopic effects provoked by the action of the Sun and the Moon on the equatorial terrestrial swelling.

At this explanation by Newton, once again artfully manipulates the various relationships among the separate actions of the Sun and the Moon so that the theoretical results coincide the said 50'', Bernoulli, Eulero and d'Alambert emphatically were opposed, ([16]) and [2] they, doing again the same calculations of Newton, reached the conclusion that the gyroscopic effect invoked by Newton was not enough to explain all of the 50'' observed. They reached to justify around 30'' a year [2], making hypothesis less ad hoc of

[15] In effects the point γ, more than being a well precise astronomic point, it is also and only the moment in which the duration of the day and the night coincide or the moment in which the Sun passes from a hemisphere to the other. From here the maintenance of a perduring geocentrism.
[16] Exceptional and unparalleled teacher in gyroscopic matters.

those formulated by Newton himself. In truth, the insuperable Englishman, often manipulated the results of its calculations to make them coincide with the experimental reality and that he did, as already mentioned, to justify the advancement of the lunar perigeal, even doubling the numerical results [5].

Obviously, once these illustrious scientists weren't anymore and without finding other effects that could be invoked for filling this big and still unresolved problem, that would have set in crisis the actual classical Celestial Mechanics and not (here it doesn't deal with 44'' second a century but of well 20" a year that were unjustifiable, and this according to the calculations of these three illustrious scientists), the thing has been started keeping silent. If besides it's observed the uncertainty with which the inertial moments of the terrestrial geoid are known, that, nowadays only, we know it to be formed from a thin afloat terrestrial crust on an incandescent fluid magma, [2], it's found that the 30" seconds go down even at about ten a year [2].

Certainly the continuous, considerable and undeniable *gravitational caress* that the Moon expounds on the terrestrial crust is not quite negligible, but seen and considered that the thin terrestrial crust literally floats on an incandescent magma, it is well possible to think the said action, further to be responsible of the showy tides of water (and of earth, we don't warn but that there are), would also be a concause of the verified derive of the continents of about 5-10 cm. a year, phenomenon that in conclusion also originates the disastrous earthquakes.

The certain fact that once South America was literally settled to the African continent (Pangea ([17])) could also be given by the mentioned action the Moon expounds on a terrestrial crust, because this last one effectuates in a day only a rotation around its axle, while our satellite employs about 27 days to complete a revolution around the Earth. Therefore we have an aquatic tide and earthling wave that, with sinusoidal law, lifts and lowers only a variable part of the thin terrestrial crust and that therefore, for evident phenomenas of work, can break and provoke some continuous leaving among the so-called continental sods, with consequent spillage of incandescent magma.

And on the subject it is to observe that, the so-called oceanic dorsals (mountainous chains under the oceans provoked by the magma escaped by the breakings of the terrestrial crust) between America and Europe, have a course that meaningfully follows the meridians and therefore these native lesions or breakings are in such a way and meaningfully orthogonal to the axle of terrestrial rotation and therefore to the gravitational action of the Moon.

If, returning to the principal matter, instead it is considered that the Sun is undeniably pushed, towards the point γ, from the various planets according to the mass of these last ones, we reach the conclusion that the defect of Newton's explanation can be filled by this effect.

Jupiter, itself, would push the Sun toward the point γ of about 52.06" a year. The other three external planets, Saturn, Uranus and Neptune, being less fast than Jupiter, they would have a refraining or accelerating action for which, in certain periods, the precession would go down to 52.06 - 6.27 - 3.4 - 2.08 = 40" that, added to those ones of Newton, they

[17] Unique, isolated prehistoric continent that with various and gigantic fractures has given then origin to the actual configuration of the terrestrial crust.

would really amount to the 50" today observed while, in other periods, it would draw taller values, as we will soon establish.
But let’s deepen these considerations.

<u>A fact is absolutely certain: the Sun cannot be considered absolutely firm, being also it attracted by the planets</u>. The point γ has therefore a function analogous to the one turn from the barycentric K: from a side it allows to measure the revolutions of the planets around the Sun and on the other the movement of the Sun around the planets themselves.

The phenomenon of the precession is normally introduced by the texts of astronomy in a completely different way, even inverted. It starts from the exposure of a probable and debatable effect then to reach the cause. And so starting to say that the axle of terrestrial rotation is not fixed in the space but that it describes a cone of precession around the vertical axle that passes for the center of the Earth and it is perpendicular to the plan of the ecliptics; this cone is represented in the preceding figures.

This involves that the North pole, projected on the celestial sky, slowly describes (or it would describe) a circumference, for which, after sometime, the star that points out the North continually changes. With this it allows to believe that the entity of the precession is measured observing as the direction of the celestial North pole varies in time, without considering the polodìa’s phenomenon and that is the continuous and sensitive move of the North pole on the terrestrial crust. In effects this polar variation is well difficult to establish, instead at difference of the easy relief of the advance with which the Sun reaches the point γ every year.

Obviously there should be a perfect correspondence among the move of the polar star and the presumed Newtonian advancement of the point γ if there could be the possibility to count on the absolute fixity of the Sun.

In every case, an accurate relief of the move of the polar star could constitute an irreplaceable help to establish as concretely the facts are. If we Want, we could also resort to the pendulum of Foucault.

7 - The planets and the absolute move of the Sun

Certainly, if considerable graves would vertically fall on the Earth from different directions, this last one would have an acceleration toward them that is the resulting of the single accelerations engraved to it from each one of these graves.

The values brought in the fourth column of the chart (1.54) represent the various arc the Sun should cross, in one hundred years, according to the considered planet, around this last one, values inferred taking for valid the theory of Newton. It's therefore evident that the real move of the Sun will be the resulting of the various shifting that every planet imposes to it, moves of the Sun toward the point γ that are orthogonals (always in the hypothesis of instantaneousness of the gravitational action) to the various conjunctions the single planets with the center of it (see arrows of the Fig. 30).

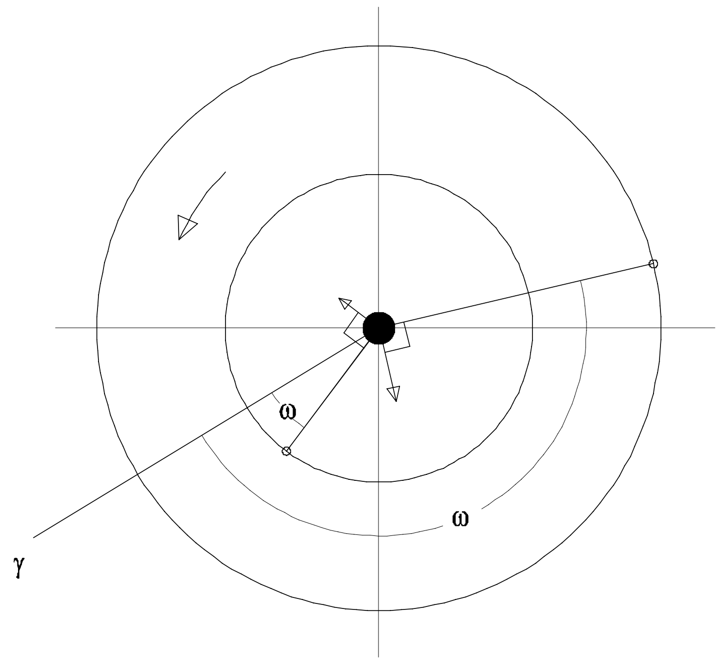

Fig. 30

If the planets, after a complete revolution around the Sun, would occupy the same reciprocal position, determined by the angles ω of the Fig. 30, then the general move of the Sun would always averagely be equal to a certain quantity and it would have an average invariable direction. Instead we know that the perhielias of the various planets, individualized by the respective angles ω(t), slowly move in the sense of revolution of the planets themselves and of different quantities.

This involves that the general move of the Sun varies both in direction and in greatness to the variation of the time. In the Fig. 24, brought ahead, the elliptic orbit of a generic planet and its perhielia is represented. From it is seen how the angle ω varies in the time. For simplicity, later, we will consider negligible the inclination of the orbit on the ecliptics, which doesn't involve big inaccuracies seen and considered that, in the case of the maximum inclination (Mercury), it is of 7° only and we will consider all the orbits to be circular because of their small eccentricity (only in the case of Mercury we have ε = 0.2). On the other hand the action of this planet on the Sun doesn't have a great weight, in fact it moves the Sun of only 44 '' a century. In every case we won't neglect this effect.

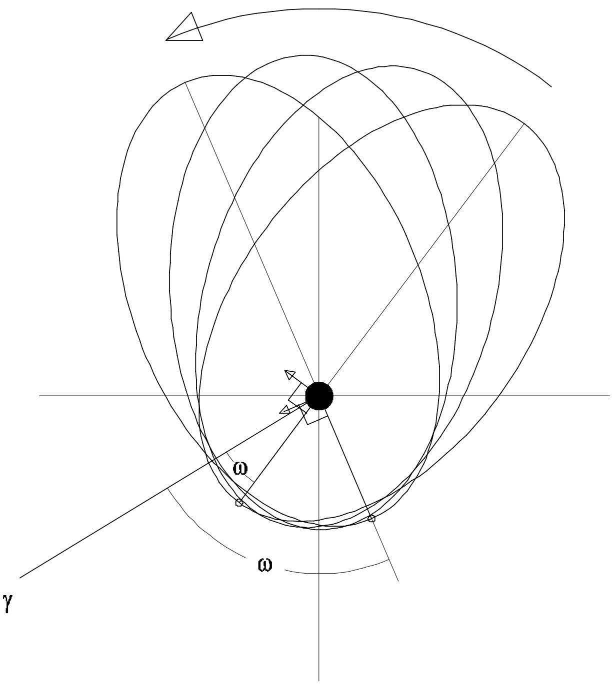

Fig. 24

The Chart (1.114), inferred by Danjon [7, pag. 430], allows to determine the positions of the various planets perihelias in comparison to the point γ, where t is calculated in Giulian centuries of 36525 days, starting from January the 1st 1900, now 0.

Pianeta	ϖ
Mercurio	$75°53'58.91" + 5599.76"t + 1.061"t^2$
Venere	$130°09'49.8" + 5068.93"t - 3.515"t^2$
Terra	$101°13'15.0" + 6189.03"t + 1.63"t^2 + 0.012"t^3$
Marte	$334°13'5.53" + 6626.73"t + 10.4675"t^2 - 0.0043"t^3$
Giove	$12°43'15.34" + 5795.862"t + 3.80258"t^2 - 0.01236"t^3$
Saturno	$91°05'53.38" + 7050.297"t + 2.9749"t^2 + 0.0166"t^3$
Urano	$171°32'55.14" + 5343.958"t + 0.8539"t^2 - 0.0128"t^3$
Nettuno	$46°43'38.37" + 5128.468"t + 1.40694"t^2 - 0.002176"t^3$

(1.114)

The chart (1.115), in the second column, brings the angle ω, varying with the time, inferred by the chart (1.114), neglecting the terms with t^2 and t^3, which does not from sensitive errors. In the third column is brought the period of necessary time so that the perihelia of the generic planet describes a complete turn around the Sun.

Pianeta	ϖ	*Periodo rivoluzione completa del perielio in anni*
Mercurio	$75°53'58.91"+5600.821"t$	23.139
Venere	$130°09'49.8"+5065.415"t$	25.585
Terra	$101°13'15.0"+6190.672"t$	20.934
Marte	$334°13'5.53"+6627.193"t$	19.556
Giove	$12°43'15.34"+5799.652"t$	22.346
Saturno	$91°05'53.38"+7053.255"t$	18.374
Urano	$171°32'55.14"+5344.809"t$	24.247
Nettuno	$46°43'38.37"+5129.873"t$	25.263

(1.115)

The said period of revolution has been calculated (for instance, in the case of Mercury) with the formula

$$T_p = \frac{360° \times 3600 \times 100}{5600.821} = 23139 \ anni\,, \tag{1.116}$$

where the value **5600.821** has been inferred by the Chart (1.115).

On the subject it is observed that the average of these periods, equal to 22 430, is of the same order of greatness of the platonic year.

To find how the general move of the Sun varies to the variation of time it, the various moves can be projected on the axle of the abscissas and the ordinates of a reference system with the origin in the center of the Sun itself (see Chart (1.33)) (inferred by the formulas of Newton) producing on it all the planets and subsequently to recompose them, as schematically is represented in the Fig. 31 (in it the perihelias are suitable with the letter π).

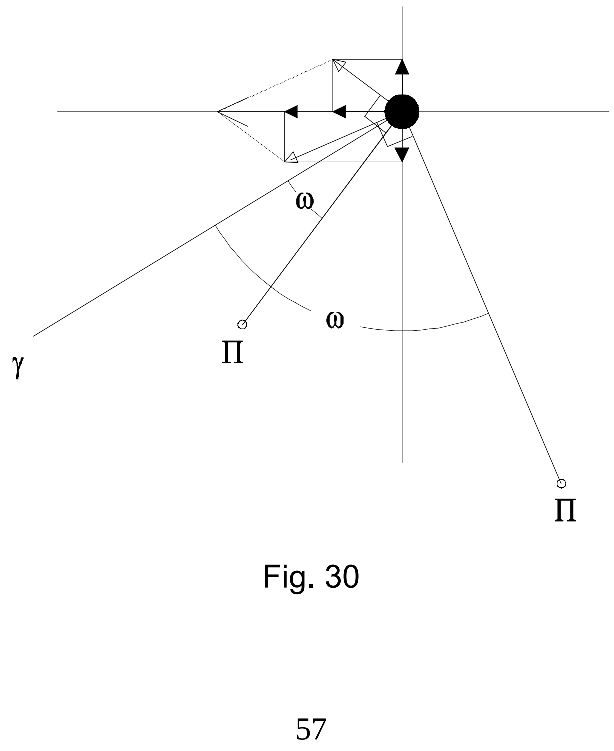

Fig. 30

It’s therefore about to calculate the resulting of vector rays rotating around the origin with different angular speeds.

The Chart (1.117) brings the necessary data for the said calculation.

Pianeta	*Spostamento del Sole in un anno*	$2\pi/T_p$
Mercurio	0.4442	$2\pi/23139.46$
Venere	2.5797	$2\pi/25\,585.27$
Terra	1.9469	$2\pi/20\,934.72$
Marte	0.1117	$2\pi/19\,555.79$
Giove	52.06	$2\pi/22\,346.17$
Saturno	6.27	$2\pi/18\,374.50$
Urano	0.3362	$2\pi/24\,247.83$
Nettuno	0.208	$2\pi/25\,263.78$

(1.117)

In the case of Mercury, the move it produces on the Sun, orthogonal to the orientation of its perihelia, projected on the axles x and y of the Fig. 30, will be given by the relations (always in this case, **1.32469** is the angle in radiants corresponding to 75°53'58.91" of the Chart (1.115))

$$x_{S_{Mercurio}} = 0.4442\cos\left(\frac{2\pi}{23139.46}t + 1.32469 - \frac{3}{2}\pi\right)$$
$$y_{S_{Mercurio}} = 0.4442\sin\left(\frac{2\pi}{23139.46}t + 1.32469 - \frac{3}{2}\pi\right), \qquad (1.118)$$

and where **(-3/2 π**) determinates the direction of the planet action on the Sun.

For Venus it will be had

$$x_{S_{Venere}} = 2.5797\cos\left(\frac{2\pi}{25585.26}t + 2.2717 - \frac{3}{2}\pi\right)$$
$$y_{S_{Venere}} = 2.5797\sin\left(\frac{2\pi}{25585.26}t + 2.2717 - \frac{3}{2}\pi\right) \qquad (1.119)$$

for the Earth

$$x_{S_{Terra}} = 1.9469\cos\left(\frac{2\pi}{20934.72}t + 1.7666 - \frac{3}{2}\pi\right)$$
$$y_{S_{Terra}} = 1.9469\sin\left(\frac{2\pi}{20934.72}t + 1.7666 - \frac{3}{2}\pi\right) \qquad (1.120)$$

for Mars

$$x_{S_{Marte}} = 0.1117\cos\left(\frac{2\pi}{19555.79}t + 0.8514 - \frac{3}{2}\pi\right)$$
$$y_{S_{Marte}} = 0.1117\sin\left(\frac{2\pi}{19555.79}t + 0.8514 - \frac{3}{2}\pi\right) \quad (1.121)$$

for Jupiter

$$x_{S_{Giove}} = 52.06\cos\left(\frac{2\pi}{22346.17}t + 0.21029 - \frac{3}{2}\pi\right)$$
$$y_{S_{Giove}} = 52.06\sin\left(\frac{2\pi}{22346.17}t + 0.21029 - \frac{3}{2}\pi\right) \quad (1.122)$$

for Saturn

$$x_{S_{Saturno}} = 6.27\cos\left(\frac{2\pi}{18374}t + 1.5899 - \frac{3}{2}\pi\right)$$
$$y_{S_{Saturno}} = 6.27\sin\left(\frac{2\pi}{18374}t + 1.5899 - \frac{3}{2}\pi\right) \quad (1.123)$$

for Uran

$$x_{S_{Urano}} = 0.3362\cos\left(\frac{2\pi}{24247.83}t + 2.994 - \frac{3}{2}\pi\right)$$
$$y_{S_{Urano}} = 0.3362\sin\left(\frac{2\pi}{24247.83}t + 2.994 - \frac{3}{2}\pi\right) \quad (1.124)$$

for Neptune

$$x_{S_{Nettuno}} = 0.208\cos\left(\frac{2\pi}{25263.78}t + 0.8155 - \frac{3}{2}\pi\right)$$
$$y_{S_{Nettuno}} = 0.208\sin\left(\frac{2\pi}{25263.78}t + 0.8155 - \frac{3}{2}\pi\right) \quad (1.125)$$

in conclusion it will be had

$$X(t) = \sum_{1}^{8} x_{S_P}(t)$$
$$Y(t) = \sum_{1}^{8} y_{S_P}(t) \quad (1.126)$$

and that the move of the Sun in function of time is equal to

$$S_{Sole}(t) = \sqrt{X(t)^2 + Y(t)^2}\,. \quad (1.127)$$

Evidently the solar move given by the (1.127) is a function that is defined in the whole inside of time $t \in [-\infty, +\infty]$. We don't know what the position of the perihelias of the planets was at the times of the formation of the solar system and therefore we don't know when the configuration given by the Chart (1.114), referring to the year 1900 A. C., is occurred for the first time, therefore the time t of the said function is known less than for a constant.

The function (1.127) is brought in the Fig. 31. It is a periodic sinusoidal irregular function whose semi-period is about 60.000 years. The program of calculation used (program Maple) is brought in appendix. It has been assumed a Compatible initial time, in such a way, with the actual situation (t=0=1900).

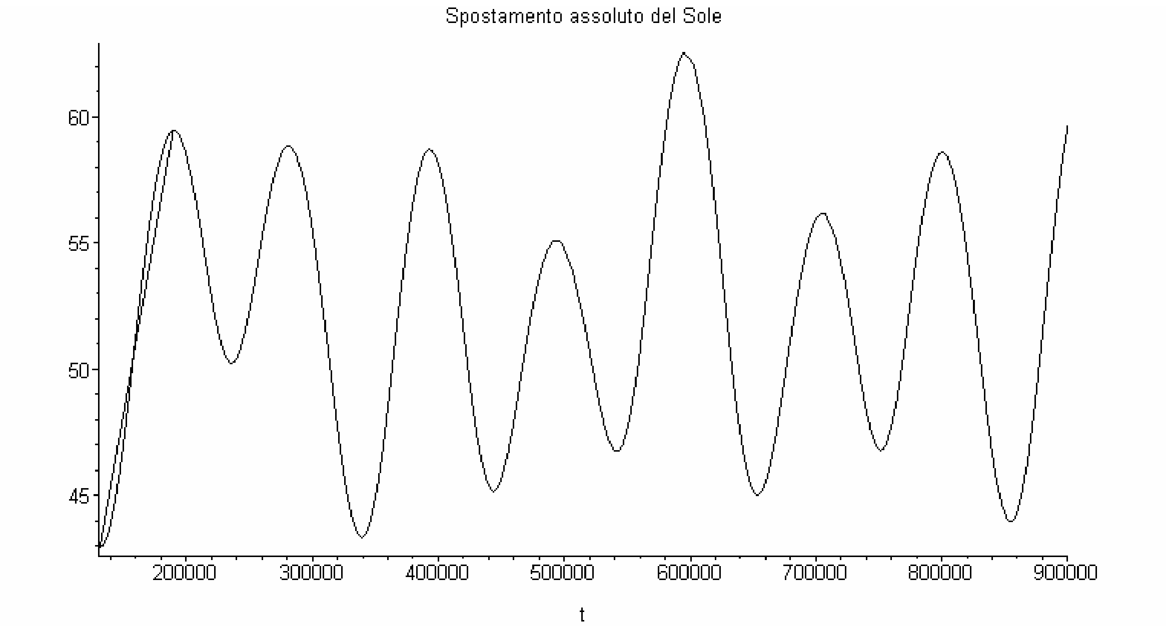

Fig. 31

It's also possible to esteem the annual average variation that suffers the said move and it results (see segment in Fig. 31; it is noticed, on the subject that these shifting segments have all the same inclination and at the same way the decreasing ones)

$$\text{var}.\,annuale = \frac{58.2-43}{189319-130996} = \frac{15.2}{58\,696.6} = \pm\,0.00026. \qquad (1.128)$$

If therefore it is assumed that currently, as it is seen by the graph, the move of the Sun produced by all the planets is equal to about 43" a year and they are attributed to the gyroscopic explanation about 10" a value of about 53" a year is had, practically very next to that one measured. According to some data of Newcomb it is had besides that the precession, in the period of observation he considered, is given by the formula

$$50.2564 + 0.000222\left(anno - 1900\right), \qquad (1.129)$$

and therefore, as it is seen, the experimental rate of growth of the precession in a year is equal to 0.000222. So a coincidence of four decimal figures is had among the theoretical value of the precession increasing (1.128) and that experimental one (1.129), variation that doesn't find any justification in the circle of the Newtonian and today's explanation.

8 - Conclusions

Let's synthetically see as in C.M. the Problem of the two Bodies is resolved. The various phases are the followings:

- It is admitted, in a first moment, that two generic masses M and m are subject to the strength of Newton and therefore they accelerate, the one verse the other, in a fixed reference system that can be identified with the one the fixed stars. In this phase it is shown, in unexceptionable way, that the barycentre K of the two masses or it is fixed or it is endowed with rectilinear and uniform motion.

- Subsequently we pass from the said reference system to the one anchored to the barycentre K, system that can be considered or in quiet with the preceding system or to be endowed with rectilinear and uniform motion, in base to the deduction of which to the first point. Both in the case that K is fixed or is endowed with rectilinear and uniform motion, which implicates that its acceleration is null, even the observer anchored to this reference system will attribute the same identical accelerations that these two masses had in comparison to the reference system considered to the first point. In fact it will add its void acceleration to the different accelerations of M and m. Therefore the law of attraction, in the passage among these two inertial systems, not only preserves its form, but it also attributes the same numerical values to the dynamics of the two masses.

Then we pass from the system K to the system anchored to the central mass M. This is a necessity caused by the fact that K is not an astronomically detectable point and that all the experimental determinations already take as reference the central mass M (see laws of Keplero). This time the observer anchored to M, in his local reference system, will attribute to the body of mass m even his non null acceleration and in addiction he will be considered fixed to all the effects. In base to this last assumption the whole theory of the secular perturbations develops and it's not then, without any procedure of homogenization and transformation of these calculations, it is pretended to find comparison among the theoretical results so gotten and the experimental reliefs that are observed instead in the first reference system and that is the one anchored to the fixed stars, forgetting to have attributed that acceleration to the peripheral mass that the central mass M had in comparison to the native reference system and to have removed it from this last one.
It's clear that this procedure, to find again favorable comparison with the reported facts to the native reference system (fixed stars), it owes to attribute again to the central mass that is the Sun, that movement that it has in comparison to the primitive reference system, characterized by the relation

$$M\ v_{Sole} = m\,V_{Pianeta}\,, \tag{1.130}$$

And therefore that procedure of ours to consider the secondary mass negligible m first and then to attribute to it its real value allows us to determine the proper move of the central mass and therefore the solar move. From the theoretical point it is besides strongly suspect the fact that a non inertial observer finds the same law that reveals an inertial observer. Perhaps a possible run to the resolution of this problem and that one of the n bodies could be a relationship of the type

$$F \simeq G\frac{M\ m}{d^{2+\frac{m}{M}+\alpha\frac{m''}{M}+\beta\frac{m'''}{M}+..+}}\,, \tag{1.131}$$

at least within our solar system.

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Program in Maple – Calculation of the move of the Sun

```
> restart:aa:=2*Pi:bb:=100:cc:=1.5*Pi:# Programma con Maple 10
> with(plots):with(plottools):
> Mesp:=44.42/bb:Meper:=aa/23139.46:
> Vesp:=257.97/bb:Veper:=aa/25585.26:
> Tesp:=194.69/bb:Teper:=aa/20934.72:
> Masp:=11.17/bb:Maper:=aa/19555.79:
> Gisp:=5206/bb:Giper:=aa/22346.17:
> Sasp:=627/bb:Saper:=aa/18374.49:
> Ursp:=33.62/bb:Urper:=aa/24247.83:
> Nesp:=20.8/bb:Neper:=aa/25263.78:
>
x:=Mesp*cos(Meper*t+1.32469+cc)+Vesp*cos(Veper*t+2.2717+cc)+Tesp*cos(Teper*t+1.7666+cc)+Masp*cos(Maper*t+5.833+cc)+Gisp*cos(Giper*t+0.222+cc)+Sasp*cos(Saper*t+1.5899+cc)+Ursp*cos(Urper*t+2.994+cc)+Nesp*cos(Neper*t+0.8155+cc):
>
y:=Mesp*sin(Meper*t+1.32469+cc)+Vesp*sin(Veper*t+2.2717+cc)+Tesp*sin(Teper*t+1.7666+cc)+Masp*sin(Maper*t+5.833+cc)+Gisp*sin(Giper*t+0.222+cc)+Sasp*sin(Saper*t+1.5899+cc)+Ursp*sin(Urper*t+2.994+cc)+Nesp*sin(Neper*t+0.8155+cc):
>
x1:=Mesp*cos(Meper*0+1.32469+cc)+Vesp*cos(Veper*0+2.2717+cc)+Tesp*cos(Teper*0+1.7666+cc)+Masp*cos(Maper*0+5.833+cc)+Gisp*cos(Giper*0+0.222+cc)+Sasp*cos(Saper*0+1.5899+cc)+Ursp*cos(Urper*0+2.994+cc)+Nesp*cos(Neper*0+0.8155+cc):
>
y1:=Mesp*sin(Meper*0+1.32469+cc)+Vesp*sin(Veper*0+2.2717+cc)+Tesp*sin(Teper*0+1.7666+cc)+Masp*sin(Maper*0+5.833+cc)+Gisp*sin(Giper*0+0.2222+cc)+Sasp*sin(Saper*0+1.5899+cc)+Ursp*sin(Urper*0+2.994+cc)+Nesp*sin(Neper*0+0.8155+cc):c1:=5.8*22500:c2:=40*22500:
> a1:=plot([x,y, t=0..20*22500],color=black):
> a2:=line([0,0], [x1,y1]):
> R:=(sqrt(x^2+y^2)):
> a3:=plot(R,t=c1..c2,color=black):
>
> display({a3,line([131566.73,42.98], [189610.58,59.48])}, title=`Spostamento assoluto del Sole`);
```