Resonance Frequencies And Planck's Formula

Brief mentions on the "quantization" of gravitational waves

Carlo Santagata
Via Giorgio Perlasca, 9
81055 S. Maria C.V. (CE) Italy
carlo.santagata@tin.it

September 13, 2005

Abstract

It's possible to show that Planck's postulate saying that $E = h\nu n$, birth certificate and fundamental platform of the actual Quantum Mechanics, is strongly deducible by **classic** theory of resonances. This is the result of a simple and intuitive experimental test, reported in the alloyed motion. In this script, after the consequent analytic form, we examine the first immediate consequences that lead us to reinterpret the actual Q. M. results, finding new ones.
The incurable fracture between macro and microcosm seems to be radically definitely removed.

1 Resonance frequencies and Planck's formula

If we consider, without attrition, a pendulum (fig. 1, alloyed picture) whom oscillation period is τ, we can amplify its amplitude if we apply to it impulses that follow with a temporal interval T able to satisfy the condition

$$T = \tau \qquad (1)$$

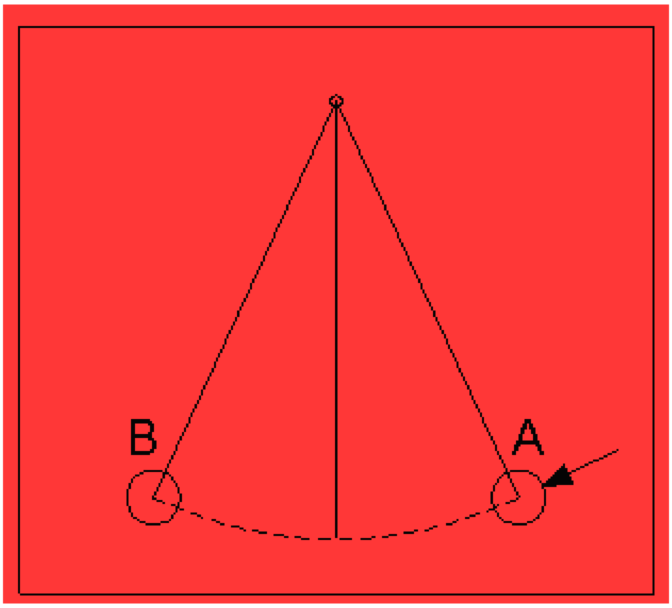

fig. 1

from what descends the known resonance condition

$$\Omega = \omega \qquad (2).$$

The solution of the classic equation

$$\ddot{x} + \gamma\dot{x} + \omega^2 x = \delta\cos(\Omega t) \qquad (3),$$

allow us to analytically describe, in a better way, the entire phenomenon. Instead, and it's useful to underline it, the (2) is only one of the many news obtainable from the complete solution of (3), that, with rigor it's neither extremely precise because the maximum ampleness does not verify, because of the presence of γ, just when (2) is fully satisfied. Infact this one is rigorously valid only when the *attrition* γ is exactly null. By these circumstantiations we observe that the (1) (or the (2)) only constitutes one of the infinites and numberable resonance conditions that can describe themselves in the daily and macroscopic reality. On the other hand it's easy and immediate to see that a most general resonance condition is given by the identity

$$T = \tau n \; (n = 1, 2, 3, ...) \qquad (4),$$

where n can be any *entire* number.

Infact, if we apply to any kind of oscillator (a pendulum, an harmonic oscillator, a dipole, a bridge, a skyscraper etc.) external impulses able to satisfy the (4), in that case only our action will never be in contrast with the said oscillator's movement, by surely provoking resonance phenomenon. And it's important to observe that only through this fundamental phenomenon any form of external energy can be absorbed by the system in exam, differently it's crossed, leaving it undisturbed or can even block it.

If τ is the period of a pendulum without attrition (see picture) and that one of our external action is exactly equal to $n\tau$,it's immediate the conviction that, *although n is rigorously and entire number*, resonance is anyway verified.

The motion alloyed to this work reports an easy experimental verification that can be done with a common pendulum. In that case the operator of the motion verifies that resonance phenomenon is present even in the case that $T = 2\tau$. Anyway it's possible to realize more sophisticated appliances than the one used in the film for more experimental verifies of the (4).

By anticipating even the analytic demonstration, we can say that a more general resonance condition is obtainable when the forcer pulsation Ω and that one of the oscillating system ω satisfy the condition

$$\omega = \Omega n \;\; (n = 1, 2, 3....) \qquad (5).$$

It's necessary to precise that the (5) is valid if the external forcer is applied everytime that pendulum mass, starting from point A, arriving to B, would come back in A, obviously without attrition. In that case the external forcer always acts in the direction $\widehat{AB}$. Instead, when it, changing alternately sign, can also act in the direction $\widehat{BA}$, the condition (5) becomes evidently

$$\omega = \frac{1}{2}\Omega n \;\; (n = 1, 2, 3....) \qquad (5bis).$$

For an harmonic oscillator without attrition, in relation to total energy, we can write

$$E_{tot} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = m\bar{v}^2 = k\bar{x}^2 \qquad (6)$$

so

$$E_{tot} = m\bar{v}^2 = m(\bar{x}\bar{\omega})^2 = m\bar{x}(\bar{x}\bar{\omega})\bar{\omega} = m\bar{x}\bar{v}\bar{\omega} \qquad (7).$$

The said oscillator will conserve indefinitely in time its oscillations. If we apply a forcer to it that satisfies the condition (5), after the transitory, we'll have

$$E_{tot} = m\bar{x}\bar{v}\bar{\Omega}n = 2\pi m\bar{x}\bar{v}\bar{\nu}n = H\bar{\nu}n \qquad (8).$$

But it's easy to verify that the action

$$Azione = H = 2\pi m\bar{x}\bar{v} \qquad (9)$$

coincides with Planck's one only in the case of an electric dipole, while the (8) is a general condition in which the action varies from case to case[1]. Infact, only in the case of the electric dipole, we have

$$Az. = 2\pi m\bar{x}\bar{v} = 2\pi m\frac{\bar{x}\bar{v}^2}{\bar{v}} = 2\pi m\frac{\bar{x}}{\bar{v}}\frac{e^2}{m\bar{x}} = \frac{2\pi e^2}{\bar{v}} \qquad (10).$$

Having on mind the known relation

$$2\pi 137e^2 = hC \qquad (11)$$

and that

$$\bar{v} = \frac{C}{137} \qquad (12)$$

we have that the (10) becomes

$$Az. = \frac{2\pi e^2}{\bar{v}} = h \qquad (13).$$

To show the intuition expressed by (5) is rigorous we can solve the following equation

$$\ddot{x} + \omega^2 x = \delta \left\{ \frac{a}{\pi} - \frac{2}{\pi} \left[\begin{array}{c} \frac{1}{1}\sin(a)\cos(\frac{\omega}{n}t) - \frac{1}{2}\sin(a)\cos(\frac{2\omega}{n}t) + \\ +\frac{1}{3}\sin(a)\cos(\frac{3\omega}{n}t) + .. \end{array} \right] \right\} \qquad (14)$$

which forcer is represented in fig. 3

[1] In an old work the author written in 1986, was shown that we can substitute to the classic ray of the electron Schwarzschild's ray and to the fine structure constant the corresponding gravitational one, we obtain Borh's corresponding equations in gravity. The Nobel Abdus Salam, at time director of the **International Centre for Theoretical Physics** of Trieste (SISSA), appreciated the manuscript and put it in the Reading Hall of the centre (see attached letter). He, as it's known, had deleted some divergences of Quantum Mechanics recurring to the non linearity of the General Relativity (even this theory has still its singularities) usable as a running bend (cutt-off) for its microscopic phenomenas, then hypothesizing the particle relieved by our Rubbia . Probably his appreciation was given by the possibility that a gravity prospected in the article could have, in versus, eliminate this last one's divergences (nowadays problem still unsolved (S. Hawking)). About gravitational waves it seeable in the note n. 3 and the last page of this script.

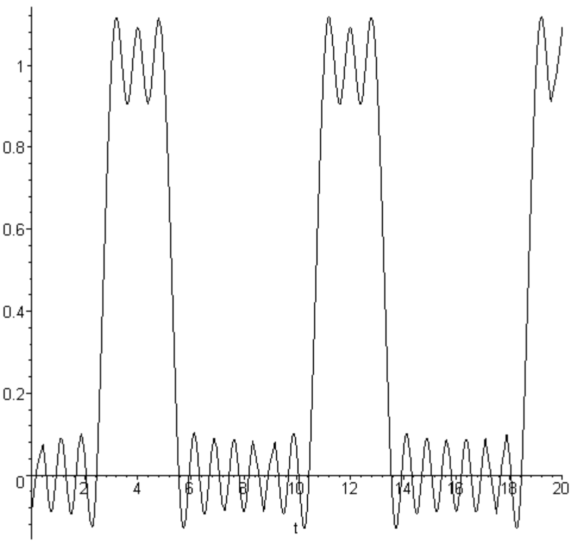

fig. 3

where a represents the little time during that is applied the said forcer. In that case temporal interval of the equation (14) with whom the external impulses are applied to the system satisfy the condition (5). The solution of (14), having stopped Fourier's develop to the fourth term, it's given by the following equation (15).

$$x(t) = \sin(\omega t)\_C2 + \cos(\omega t)\_C1 + \frac{1}{3}\Bigg((84n^4 - 64n^2 + n^8 - 21n^6)\sin\left(\frac{3(\omega t - an)}{n}\right) + \left(-\frac{147}{4}n^4 + \frac{21}{2}n^6 - \frac{3}{4}n^8 + 27n^2\right)\sin\left(\frac{4(\omega t - an)}{n}\right) + \left(-\frac{3}{2}n^8 + 216n^2 - \frac{507}{2}n^4 + 39n^6\right)\sin\left(\frac{2(\omega t - an)}{n}\right) + (-87n^6 + 732n^4 + 3n^8 - 1728n^2)\sin\left(\frac{\omega t - an}{n}\right) + (-84n^4 - n^8 + 21n^6 + 64n^2)\sin\left(\frac{3(\omega t + an)}{n}\right) + \frac{3}{4}\Bigg((n^6 - 5n^4 + 4n^2)\sin\left(\frac{4(\omega t + an)}{n}\right) - 4(-4+n)(4+n)\Bigg(\left(\frac{1}{2}n^2 - \frac{1}{2}n^4\right)\sin\left(\frac{2(\omega t + an)}{n}\right) + (-2+n)(2+n)\left(n^2\sin\left(\frac{\omega t + an}{n}\right) - an^2 + a\right)\Bigg)\Bigg)(-3+n)(3+n)\Bigg)\delta \Big/ \left(\pi\omega^2(576 - 820n^2 + 273n^4 - 30n^6 + n^8)\right)$$

The study of this solution leads to the conclusion that when n, by growing with absolute continuity, has entire values $1, 2, 3$...the third term numerator never annuls while its denominator that is the equation

$$A = \pi\omega^2\left(576 - 820n^2 + 273n^4 - 30n^6 + n^8\right) = 0 \qquad (16)$$

is equal to zero only when

$$n = \pm 1, n = \pm 2, n = \pm 3, n = \pm 4,$$

end so oscillator ampleness becomes infinite just when n has the said values. The figures 4 and 5

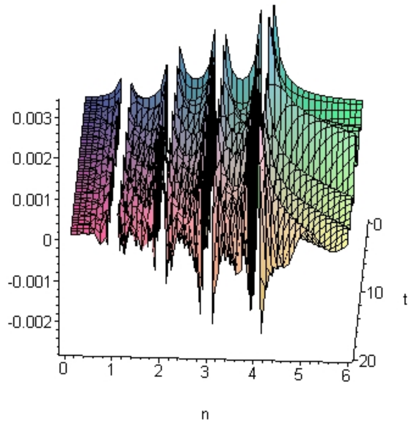

fig. 4

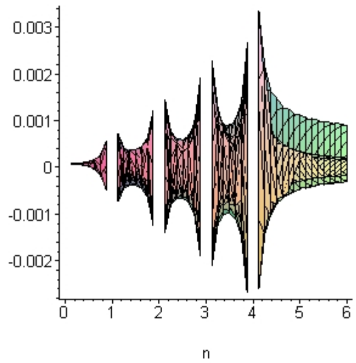

fig. 5

graphically represent the third term of the equation (15) (stationary solution), obtained making vary with the more absolute continuity the variable n.

Always without damping, in the general case, and so when we consider innumerable terms of Fourier's series, we find that (16) is

$$\left(n^2-1\right)\left(n^2-2^2\right)\left(n^2-3^2\right)\left(n^2-4^2\right)..\left(n^2-m^2\right)....=0.$$

So in conclusion that the intuition expressed by (5) is fully confirmed and that Planck non-shown equation only constitutes a macroscopic and more ample resonance condition comparable in the daily reality. It can't be different, infact radiation absorption from part of material can only happen through resonance phenomenon. In every case is to observe the Planck's empiric formula is able to so on this phenomenon the tip of an iceberg so it cannot explain it in its total completion.

The more complex equation remains to be solved

$$\ddot{x}+\gamma\dot{x}+\omega^2 x=\delta\left\{\frac{a}{\pi}-\frac{2}{\pi}\left[\begin{array}{c}\frac{1}{1}\sin(a)\cos(\frac{\omega}{n}t)-\frac{1}{2}\sin(a)\cos(\frac{2\omega}{n}t)+\\ +\frac{1}{3}\sin(a)\cos(\frac{3\omega}{n}t)+..\end{array}\right]\right\} \tag{17}$$

its solution is not synthetic so it's more difficult to manage. By stopping Fourier's series at the second term we have the stationary solution

$$
\begin{aligned}
\mathrm{x}(t) = -\frac{1}{2}\Bigg(& \frac{1}{16}\left(\frac{(n-1)^2 (n+1)^2 \omega^2}{4} + n^2 g^2\right) n^2 (n+2)(n-2)\,\omega^2 \\
& \sin\left(\frac{2(-a n + \omega t)}{n}\right) \\
& - \frac{1}{4}\left(\frac{(n-1)^2 (n+1)^2 \omega^2}{4} + n^2 g^2\right) n^3 g\,\omega \cos\left(\frac{2(-a n + \omega t)}{n}\right) - \frac{1}{2} \\
& n^2 \left(\left(-\frac{1}{2} n^2 + 1 + \frac{1}{16} n^4\right)\omega^2 + n^2 g^2\right)(n-1)\,\omega^2 (n+1) \\
& \sin\left(\frac{-a n + \omega t}{n}\right) \\
& + n^3 \left(\left(-\frac{1}{2} n^2 + 1 + \frac{1}{16} n^4\right)\omega^2 + n^2 g^2\right) g\,\omega \cos\left(\frac{-a n + \omega t}{n}\right) - \frac{1}{16} \\
& \left(\frac{(n-1)^2 (n+1)^2 \omega^2}{4} + n^2 g^2\right) n^2 (n+2)(n-2)\,\omega^2 \\
& \sin\left(\frac{2(a n + \omega t)}{n}\right) \\
& + \frac{1}{4}\left(\frac{(n-1)^2 (n+1)^2 \omega^2}{4} + n^2 g^2\right) n^3 g\,\omega \cos\left(\frac{2(a n + \omega t)}{n}\right) - \\
& \Bigg(\left(\left(-\frac{1}{2} n^2 + 1 + \frac{1}{16} n^4\right)\omega^2 + n^2 g^2\right)\Bigg(\left(-\frac{1}{2} n^4 + \frac{1}{2} n^2\right)\omega^2 \sin\left(\frac{a n + \omega t}{n}\right) \\
& + \omega \cos\left(\frac{a n + \omega t}{n}\right) n^3 g + 2\left(\frac{(n-1)^2 (n+1)^2 \omega^2}{4} + n^2 g^2\right) a\Bigg)\Bigg)\Bigg)\,\delta \\
& \Bigg/ \Bigg(\pi \left(\left(-\frac{1}{2} n^2 + 1 + \frac{1}{16} n^4\right)\omega^2 + n^2 g^2\right) \\
& \left(\frac{(n-1)^2 (n+1)^2 \omega^2}{4} + n^2 g^2\right)\omega^2\Bigg)
\end{aligned}
$$

graphically represented by fig. 6.

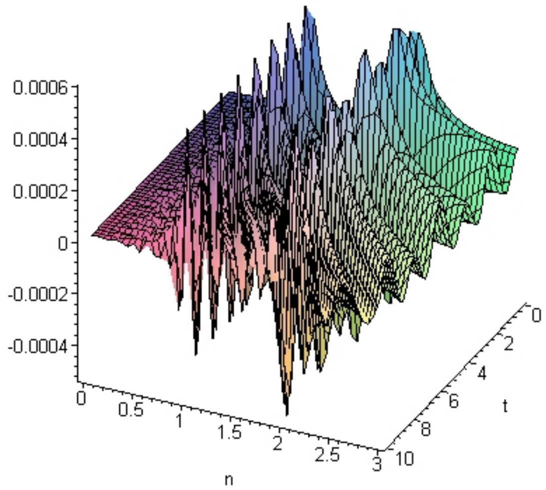

fig. 6

In the case of a Fourier develop stopped at the third term we have the fig. 7

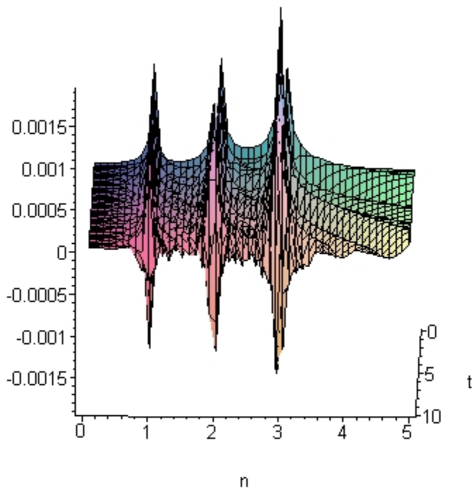

fig. 7

An analysis of these equations class leads to the conclusion the amplitude resonance is represented by the following equations (fig. 8-9)

$$\Psi = \frac{\delta f(sen(\omega), \cos(\omega))}{\left((n^2-1^2)^2+n^2\gamma\right)\left((n^2-2^2)^2+n^2\gamma\right)\left((n^2-3^2)^2+n^2\gamma\right)...} \qquad (18)$$

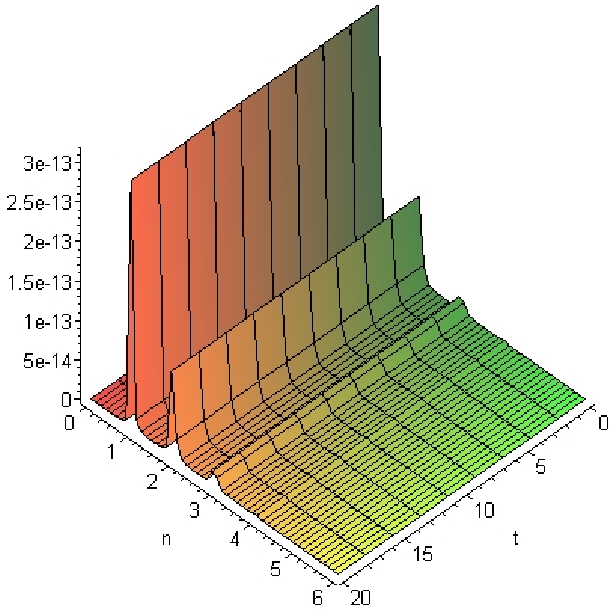

fig. 8

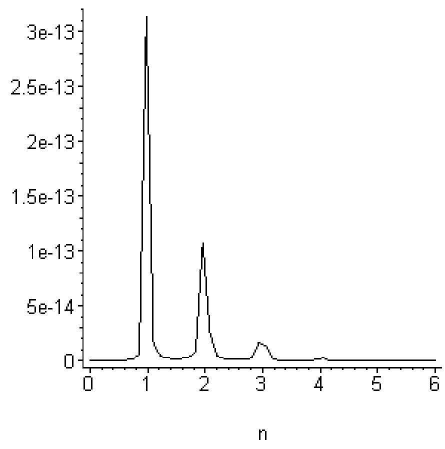

fig. 9

The (18) can be written in the following way

$$\begin{aligned}\Psi &= \frac{\delta f(sen(\omega),\cos(\omega))}{\left((n^2-1^2)^2+n^2\gamma\right)\left((n^2-2^2)^2+n^2\gamma\right)....} = \\ &= \frac{\delta f(sen(\omega),\cos(\omega))}{\prod\limits_{m=1,2,3,...}^{m=n}\left((n^2-m^2)^2+n^2\gamma\right)} \qquad (19)\end{aligned}$$

2 Resonance energy of the hydrogen atom

The equation's solutions (17) are not easy manipulable even because of the presence of some undesirable terms caused by the approximations of a Fourier series develop. We can provisionary utilize classic equation solutions (3), taking opportunely present the resonance condition given by (5).

In the case of energy the solution of the (3) gives the known relation

$$E_{ris} = \frac{1}{2}m\Omega^2\frac{\delta^2}{(\Omega^2-\omega^2)^2+\gamma^2\Omega^2} \qquad (20).$$

When verifies the condition that $\omega = n\Omega$ we have

$$E_{ris} = \frac{1}{2} m \frac{\delta^2}{\gamma^2} \qquad (21)$$

and the oscillator has assumed the same pulsation of the forcer that is $\Omega = \frac{\omega}{n}$.

By indicating with $\bar{v}$ its medium velocity, in resonance conditions, we can write

$$E_{ris} = \frac{1}{2} m \frac{\delta^2}{\gamma^2} = \frac{1}{2} m \frac{(\bar{v}\Omega)^2}{\gamma^2} \qquad (22).$$

Taking count of (5), the (22) becomes

$$E_{ris} = \frac{1}{2} m \bar{v}^2 \left(\frac{\Omega}{\gamma}\right)^2 = \frac{1}{2} m \bar{v}^2 \left(\frac{\omega}{\gamma}\right)^2 \frac{1}{n^2} \qquad (23),$$

that has the same structure of the relation given by Q.M., obtained by the hypothesis of the elementary quantum of energy . To let (23) coincides with experimental facts we must pose

$$\frac{\bar{v}\omega}{\gamma} = \frac{C}{137} \qquad (24).$$

Infact, in that case the (23) becomes

$$E_{ris} = \frac{1}{2} \frac{mC^2}{137^2} \frac{1}{n^2} \qquad (25),$$

that, taking count of (11), coincides with the known results.

3 Calculation of resonance amplitude

From the known relation related to resonance amplitude we have

$$A = \Psi = \frac{\delta}{\sqrt{(\Omega^2 - \omega^2)^2 + \gamma^2\Omega^2}} \qquad (26).$$

It follows that for $\omega = n\Omega$

$$A = \Psi = \frac{\delta}{\gamma\Omega} \qquad (27)$$

or, taking count of the position (24) and of the (5), we have

$$A = \Psi = \frac{\bar{v}\Omega}{\gamma\Omega} = \frac{\bar{v}}{\gamma} = \frac{C}{137\omega} = \frac{C}{137\Omega n} = \frac{C}{2\pi 137\nu n} = \frac{\lambda}{2\pi 137 n} \qquad (28)$$

from the previous relation descends that amplitude vibration of the charge is alloyed to the electromagnetic wave length of the new relation[2]

$$\lambda = 2\pi 137 \Psi n \qquad (29)\,.$$

This result has just been found in the work [1].

4 Bohr's results

By multiplying both the members of the (29) for the frequency we have

$$\lambda\nu = 2\pi 137 \Psi \nu n = C \qquad (30)$$

from what

$$137\Psi\Omega n = 137\bar{v}n = C \qquad (31)$$

so

$$\bar{v} = \frac{C}{137n} \qquad (32)$$

or

$$E_{ris} = \frac{1}{2}m\bar{v}^2 = \frac{1}{2}m\frac{C^2}{137^2}\frac{1}{n^2} \qquad (33)\,,$$

result already obtained.

Because of

$$\bar{v} = \sqrt{\frac{e^2}{m\Psi}} \qquad (34)\,,$$

by equalizing (32) and the (34), we obtain that

[2] For a gravitational dipole we have

$$\lambda_g = 2\pi\frac{\bar{C}}{\bar{v}}\psi n$$

where $\bar{C}$ is the propagation velocity of gravitational waves, $\bar{v}$ and ψ are the medium velocity and the (secondary) mass amplitude of the said dipole.

$$\Psi = \frac{e^2}{mC^2} 137^2 n^2 \qquad (35)$$

a relation that, taking count of (11), coincides with that already known one and so with Borh's rays. So resonance ampleness coincides with Borh's particular rays.

5 The new bond between de Broglie's wave and the electromagnetic wave

Because the wave λ_{dB} of de Broglie is alloyed to Bohr's orbits from the known relation

$$\lambda_{dB} = 2\pi \frac{e^2}{mC^2} 137^2 n^2 \qquad (36),$$

having on mind the (11), we also have

$$\lambda_{dB} = \frac{hC}{mC^2} 137 n^2 = \frac{h}{m\frac{C}{137n}} n = \frac{h}{m\bar{v}} n \qquad (37).$$

But the (36) can also be written

$$\lambda_{dB} = 2\pi \frac{e^2}{\frac{mC^2}{137^2 n^2}} = \frac{2\pi e^2}{m\bar{v}^2} \qquad (38)$$

and, taking count of (34), we have

$$\lambda_{dB} = \frac{2\pi e^2}{m\bar{v}^2} = 2\pi\Psi \qquad (39).$$

Having on mind the relation

$$\lambda = 2\pi 137 \Psi n \qquad (29)$$

the (39) permits to alloy de Broglie's wave length λ_{dB} with the electromagnetic one λ through the new relation

$$\lambda = \lambda_{dB} 137 n \qquad (40)\,.$$

All of this for the alloyed atom. For the ionized dipole we'll have

$$\lambda_{dB} = \frac{h}{m\bar{v}} \qquad (41),$$

$$\lambda = 2\pi 137\Psi \qquad (42)$$

and

$$\lambda = \lambda_{dB} 137 \qquad (43).$$

6 Remark

The (43) says that the charge follows the same laws of the electromagnetic radiation, except for the constant 137. Infact it can be differently because its cinematic is entirely dictated by resonance phenomenon, that can verify only for alloyed charges. When the charge is not alloyed it assumes the famous corpuscular behavior. So is totally interpreted to Principia of Complementary (Bohr 1928) for which cannot exist an experiment able to contemporaneously disclose undulated and corpuscular ambient of material.

7 Glances about the gravitation waves quantization

If it's correct the said interpretation of the enigmatic equation by Planck for that the relation

$$E = \hbar\omega = \hbar\Omega n \qquad (44)$$

is nothing else that a more complete resonance condition deducible in the daily and macroscopic reality, for a gravitational dipole we can analogously write

$$\lambda_g = 2\pi \frac{\bar{C}}{\bar{v}} \Psi n \qquad (45)$$

where $\bar{C}$ is the propagation velocity of the gravitational waves, $\bar{v}$ is the medium velocity of the dipole, ψ is its amplitude and n is an entire number that assures the resonance condition between the wave and the dipole itself.

In the case of the dipole Jupiter-Io, in the hypothesis that $\bar{C}$ coincides with light's velocity, we have that

$$\lambda_g = 2\pi \frac{\bar{C}}{\bar{v}} \Psi n = \frac{2\pi C}{\sqrt{\frac{GM}{\Psi}}} \Psi n = 4.56 \times 10^{15} n \ [cm] = 4.58 \times 10^{10} n \, [Km] \qquad (47),$$

length that is about 8 times bigger than the distance Sun-Pluto.

It should be interesting to study gravitational waves together with the known phenomenas of the reciprocal **resonances** that the various moons of Jupiter endure among them. In these circumstances we have gravitational masses that endure acceleration variations.

In analogy with the electromagnetic phenomenas we should think about a gravitational black body and how this energy is absorbed and emitted by the same one etc. etc..

References

[1] Carlo Santagata **New Quantum Relations**