From Coulomb to Yukawa through Planck
On the unification of known interactions
The resolution of the classic singleness
Abstract

Perhaps it’s possible to reconduct the mesonic interaction of Yukawa to the coulombian field in basis to unknown implications of the Black Body Problem proposed at that time by Planck.

As it’s immediate to show, the mentioned solution of Planck eliminates the innate and congenital singleness of which Coulomb’s formula (Newton) is affected [c.g.s.]

\[ F = \frac{e^2}{\psi^2} \]

for \( \psi \to 0 \), where \( e \) is the electron charge and \( \psi \) is the average distance between the two charges. It will be immediate to deduce from Planck’s relations according to the coulombian force that

\[
F = \frac{\partial E}{\partial \psi} = \frac{e^2}{\psi^2} \left[ 1 - \exp \left( \frac{e^2}{\psi mC^2} \right) \exp \left( \frac{e^2}{\psi mC^2} \right) - 1 \right]
\]

having a course deducible from the following graph.

In it we can see ho Coulomb’s force extends to infinite, by extending \( \psi \to 0 \), while the force law deducted by Planck’s solution, at first reaches the maximum, and then it entirely annuls. For big distances it extends to Coulomb’s macroscopic formula.

So we immediately deduce that the total work of annihilation of the couple of charges is equal to
$E = m\vec{C}^2$

where $\vec{C}$ can also coincide with light's speed, but, it's generally different from it. Besides from these new relations we draw that joining energy is given by the relation

$$E = \frac{e^2}{\psi} \exp\left(\frac{e^2}{\psi m C^2}\right) - 1 \approx \frac{e^2}{\psi} \exp\left(-\frac{R}{\psi}\right),$$

that, by neglecting -1, is simile to the formula hypothesized by Nobel Yukawa in the case of the nuclear forces, by having posed $e^2 / m C^2 = R$. 
Premise

Let's consider a Planck's non-polarized energy packet

\[ E = h \nu n \]  \hspace{1cm} (1.1)

that it's about to strike the Black Body's wall. The said packet to be entirely absorbed from an electromagnetic dipole constituted by a simple couple proton-electron. According to the Energy Conservation Law it will be

\[ E = h\nu n = \frac{e^2}{\psi_2} - \frac{e^2}{\psi_1}. \]  \hspace{1cm} (1.2)

we assume \( \psi_1 = \infty \), and we also have, more simply

\[ E = h\nu n = \frac{hC}{\lambda} n = \frac{e^2}{\psi}. \]  \hspace{1cm} (1.3)

by having on mind that fine constant structure, the electron charge, Planck's constant and light's speed are alloyed by the known relation

\[ 2\pi 137 e^2 = hC, \]  \hspace{1cm} (1.4)

from (1.3) immediately comes down the new relation

\[ \lambda = 2\pi 137 \psi n \]  \hspace{1cm} (1.5)

that alloys the electromagnetic wave length \( \lambda \) to the average vibration amplitude of the dipole \( \psi \).

The said relation is not in contrast with the actual Quantum Mechanics. By the way, and we limit to this brevity, from (1.5) it's immediate to deduce the energetic levels of the hydrogen atom. In fact the (1.5) can be written

\[ \lambda \nu = C = 2\pi 137 \psi \nu n = 137 \omega \psi n = 137 \nu n \]  \hspace{1cm} (1.6)

so we have that the average velocity of the charge is given by

\[ \nu = \frac{C}{137 n} \]  \hspace{1cm} (1.7)

from what follows the dipole's energy is equal to

\[ E = \frac{1}{2} \frac{mC^2}{137^2} \frac{1}{n^2} \]  \hspace{1cm} (1.8)
coinciding, minding the (1.4), with the known Bohr’s relation.

So we have that the elementary quantum of electromagnetic energy can be written as this identity

\[ E = h\nu = \hbar \frac{C}{\lambda} = \frac{e^2}{\psi}. \]  

(1.9)

The relation (1.9) can easily be deduced the following way

\[ E = \frac{e^2}{\psi} = \frac{e^2}{\psi} \frac{2\pi 137}{2\pi 137} = \frac{h C}{2\pi 137\psi} \]  

(1.10)

so it coincides with the relation (1.9) if we pose

\[ \lambda = 2\pi 137\psi. \]  

(1.11)
Implications and consequences

Even if the Problem of the Back Body can find different solutions from the one proposed by Planck [1], in this case we will use the known experimental relation

\[
E_{\lambda} = \frac{hC^2}{\lambda^{-3}} \frac{1}{\exp\left(\frac{hC}{kT\lambda}\right) - 1} \tag{1.12}
\]

that Planck was able to deduct back by the hypothesis expressed from (1.1)\textsuperscript{1}.

From it, for a generic electromagnetic dipole, we have that energy is

\[
E = \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1} = \frac{hC}{\lambda}. \tag{1.13}
\]

If we consider the (1.9) that can be posed

\[
kT = m\overline{C}^2, \tag{1.14}
\]

the (1.13) can also be written

\[
E = \frac{hC}{\lambda} \frac{1}{\exp\left(\frac{hC}{kT}\lambda\right) - 1} \frac{e^2}{\psi} = \frac{e^2}{\psi}, \tag{1.15}
\]

by having posed

\[
R = \frac{e^2}{m\overline{C}^2}. \tag{1.16}
\]

From the (1.15) it’s possible to deduct that the Coulombian average force practiced between the two charges to any distance $\psi$.

In fact we have

\[
F = -\frac{\partial E}{\partial \psi} \tag{1.17}
\]

so we obtain

\textsuperscript{1} The (1.1), despite the efforts of Planck himself, has never had a justification in the classic physics circle. To have a clear Theoretical deduction, in accord to C.M. canons, you can consult the website www.carlosantagata.it. In fact it’s referable to a more extended and complete condition of classic resonance between the forcer and the harmonic oscillator.
\[
F = -\frac{e^2}{\psi^2} \left[ \frac{1}{\exp\left(\frac{R}{\psi}\right) - 1} + \frac{\exp\left(\frac{R}{\psi}\right)}{\left[\exp\left(\frac{R}{\psi}\right) - 1\right]^2} \right]. \tag{1.18}
\]

The (1.18) is shown in the Fig. 1.

In this graph is represented, by points, Coulomb's law too. It's evident the strict connection between Rayleigh & Jeans Ultraviolet Catastrophe and the congenital singleness of the classic physics for \(\psi \to 0\), that has not still completely resolved (Nobel Abdus Salam [4]).

From the (1.18) it's possible to calculate the annihilation or separation work of a proton and an electron or other charge systems

\[
E = \left[ \frac{\frac{e^2}{\psi}}{\exp\left(\frac{e^2/\psi}{mC^2}\right) - 1} \right]_{\psi=0}^{\psi=\infty} = m \overline{C}^2, \tag{1.19}
\]
or (2)

\[
E = \left[ \frac{e^2}{\psi} \frac{e^{-R/\psi}}{\exp\left(\frac{R}{\psi}\right) - 1} \right]_{\psi=\infty} \approx \frac{e^2}{R}. \tag{1.20}
\]

From the (1.15), representing the coulombian potential, we obtain, by neglecting \(1^{-1}\),

\[
E = \frac{e^2}{\psi} \frac{e^{-R/\psi}}{\exp\left(\frac{R}{\psi}\right) - 1} \frac{e^2}{\psi} \exp\left(-\frac{R}{\psi}\right) = e^2 \exp\left(-\frac{R}{\psi}\right) \tag{1.21}
\]

That is simile to Yukawa’s potential [2, 3] for the description of nuclear forces so

\[
E = -g \frac{\exp(-\mu r)}{r}. \tag{1.22}
\]

Are we perhaps close to a unification of coulombian forces with nuclear ones?

Bibliography


2 Generally, for any number \(Z\) of charges we have

\[
E = Z \, m \, \overline{C}^2.
\]