Eppur si muove (nevertheless, it does move), but this time it is the Sun,
or
the heaviest body falls most quickly,
or
Mercury moves the Sun towards the γ point (precession) by 44'' per century,
and, for the same physical reason,
Jupiter moves it by 50'' per year, a value exactly equal to the value due to the lunisolar precession!

FIRST PART

www.carlosantagata.it   info@carlosantagata.it

Abstract

A careful study of the PRINCIPIA [1][2][3][4][5] brings us to an extraordinary conclusion: Newton, without realizing the fact (and all of us with him), makes a correction to the law of falling bodies by Galilei! By means of his formulas we find immediately that, near the surface of the Earth, a heavy body would fall quickly than a light body, but the difference in time, easy to find theoretically, is so small that, even today, it cannot be detected experimentally.

On the other hand, even if Newton's forecast on the fall of bodies (and, thus, in the restricted terrestrial environment) are still unreachable, their implications in the solar system scale are really exceptional and extraordinary.

We will show that Le Verrier and Newcomb made an error, both in concept and in calculations, concerning the motion of Mercury's perihelion. This enables us to highlight (using only the power of Newton's equations) the fundamental and always neglected role of the action of the gravitational mass of a generic planet onto the Sun. This action actually is an induced movement of the Sun, always considered absolutely fixed, that causes a slippage, towards the γ point, that is directly proportional to the gravitational mass of the orbiting body.

Making use of incontrovertible and elementary physical considerations, we will discover that, while Mercury moves the Sun towards the γ point by 44'' per century, Jupiter - the giant of the solar system - moves the Sun by 52'' per year, and the Earth by only 2'' per year. We immediately deduce a fact of exceptional importance: the grand, concrete and undeniable astronomical phenomenon known today as *lunisolar* precession, never clearly interpreted, is not due to the own movement of the γ point towards the Sun - considered absolutely fixed according to the qualified and debatable *ad hoc* version of Newton - but instead should be ascribed for a non negligible portion, to the real motion of the Sun towards the γ point. This motion is due, as already stated, to the gravitational movement that all planets induce, in various magnitudes, onto the Sun. Thus the difference between the 52'' caused by Jupiter and the 2'' caused by the Earth, that is exactly equal to the 50'' of the actual *lunisolar* precession, no more appears to be a coincidence. In other terms, as we will specifically establish with exact calculations, the terrestrial or local effect of the inaccuracy of Galilei's law becomes, in the astronomical domain, both an interpretation of the unexplainable and slight movement (precession) of Mercury's perihelion and a new

and clear reformulation of the undeniable and concrete precession, that today is named lunisolar precession. Therefore the criticism of Bernoulli, Euler and d'Alembert is true…

1. The falling bodies of Galileo Galilei

The definition of the exact way in which bodies fall on the Earth surface is a scientific question whose origins are lost in the past centuries, and for which, still today, there isn't a clear and ultimate answer. If we ask now to common people what object would fall first on Earth surface, of course in an environment without air, they certainly will answer that a tank reaches the Earth surface before a feather. And this is not the first time that the science mortifies the common sense: a list of similar cases would be long. But we will see that, at least in this case, this disqualified common sense should be revalued.
For the sake of truth, the great philosopher Plato already thought in a different way: he believed that the heaviest body would fall first.
After two thousands years we find Galileo Galilei, whose experimental laboratory - let us emphasize this point immediately - was badly equipped. To detect the times of a given phenomenon he often used his pulse[1]. Anyway, he observed (with the inclined planes he adopted to make slower the fall of the spheres used in his experiments) that the difference between the fall times of a solid sphere and a hollow sphere was not appreciable, at least at first sight. And, probably, these rudimental experiments gave him the conviction that a possible fall experiment carried out on top of Pisa tower (that, according to many experts, was never really done) should have fully supported such a deduction. Being conscious of the inaccuracy of his experiments, he imagined other experiments, never realized (for example a heavier body connected to a lighter body), that should convince himself[2] that the fall law was invariable, irrespective of the weight of falling bodies.
However, it is important to precise that, at the state of the art, the forecasts done by official science about this experiment are only of purely experimental nature, or at least all the people think so. On the contrary, we will see that Newton's theory, never invoked in this case[3], allows us to make precise forecasts that could serve as guide to design clarifying experiments.

2. Newton's gravitation

According to the Universal Gravitation Theory of Newton, two bodies at distance d, having respectively masses[4] M and m, both experience the force[5]

$$F = G\frac{Mm}{d^2} \tag{1.1}$$

and this is true **either for an observer anchored to the fixed stars or for an observer anchored to the baricentre of the two masses (even inertial, of course)**.

[1] In such a way he discovered the isochronism of small oscillations.
[2] Evidently he was the first person that was not able to accept this.
[3] This missing application can be explained because Newton, as previously stated, never realized that, correcting the 3rd Kepler's law, he automatically corrected even Galilei's law, as we will see in detail.
[4] According to Newton, the mass of a body is the quantity of matter by which it is constituted.
[5] G is the gravitational constant.

An error that is very easy to do[6] when formula (1) is used, is the following. If we want to calculate the acceleration with which a body having mass m would fall on the Earth surface[7] we can divide both members of (1) by m and obtain

$$a = \frac{F}{m} = G\frac{M}{R^2} = g. \tag{1.2}$$

This formula immediately shows that the acceleration depends on the mass of the Earth and not on the mass of the falling bodies. So we find the erroneous conclusion that, even according to Newton, all the falling bodies have the same acceleration, given by (2).

Actually, relation (2) does not give the acceleration measured by an observer on the Earth's surface. As already stated, it gives the acceleration measured by an observer anchored to the fixed stars or to the barycentre[8], which strongly depends on the masses M and m considered.
Instead, to calculate exactly what would be seen by an observer anchored to the Earth's surface we need the following argument, due to Newton (two-body problem[9]) [1].
Actually relation (1), as already noted, gives the force acting on the two bodies, measured by an observer anchored to the fixed stars or to the barycentre of the two masses. In the following, for conciseness, we will refer only to the latter.
First of all (Fig. 1) it is easy to calculate the position of the barycentre in terms of the positions of the two masses.

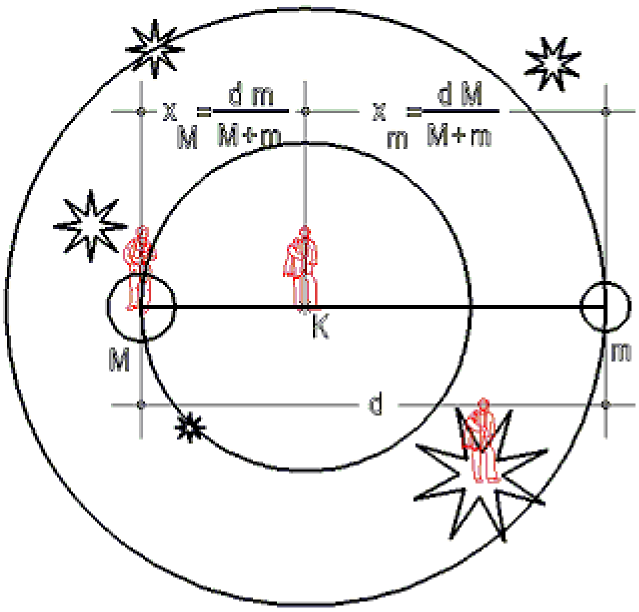

Fig. 1

Obviously we have

[6] This implies the widespread and erroneous conviction of the absolute consistency between Galilei's law and Newton's theory.
[7] Having radius equal to R.
[8] Very uncomfortable position, since this baricentre is almost coincident with the centre of the Earth.
[9] The two-body problem is very important even because the baricentre of the masses, being an abstract entity, obviously is not accessible to astronomical observations. This implies that the trajectory of a generic body having mass m is always referred to the non-inertial reference frame anchored to the central mass (heliocentric system).

$$x_M = \frac{d}{M+m} m \tag{1.3}$$

$$x_m = \frac{d}{M+m} M \tag{1.4}$$

Now, if the two masses undergo the same force given by (1), and revolve with the same angular speed around K, we will have two obvious identities[10] (equality of gravitational and centrifugal force)

$$\frac{MV_{MK}^2}{x_M} = G\frac{Mm}{d^2} \tag{1.5}$$

$$\frac{mV_{mK}^2}{x_m} = G\frac{Mm}{d^2}. \tag{1.6}$$

From these two relations we can calculate the speeds that the masses concerned have in the inertial reference frame anchored to the barycentre K

$$V_{MK} = \sqrt{\frac{Gm^2}{d\left(M+m\right)}} \tag{1.7}$$

$$V_{mK} = \sqrt{\frac{GM^2}{d\left(M+m\right)}}. \tag{1.8}$$

It is evident that the observer anchored to the mass M, i.e. to the non-inertial reference frame formed by three Cartesian axes in which that mass is at rest[11], will assign the total speed

$$V_{mM} = V_{MK} + V_{mK} = \sqrt{\frac{GM}{d}\left(1+\frac{m}{M}\right)} \tag{1.9}$$

to the mass m. Obviously, from (5) and (6) we also find the accelerations that the masses have for an observer at rest in K. They are

$$a_{MK} = \frac{Gm}{d^2} \tag{1.10}$$

[10] Here we consider only the case of circular orbits, but we will see that this choice does not restrict the validity of our conclusions in any way.

[11] Let us imagine a sphere having a given radius and a reference frame, formed by three mutually orthogonal axes, whose origin is the centre of the sphere and whose axes are strictly anchored to the surface of the sphere. This clarification is necessary because the heliocentric reference frame adopted in Astronomy has the origin in the centre of the Sun while its axes are directed towards the fixed stars. The first axis joins said centre and the γ point (fixed stars), the second axis lies in the ecliptic plane, and the third axis is orthogonal to this plane. Thus there is a substantial difference between this reference frame and the reference frame anchored to the mass M. These frames are equivalent only if both the Sun and the γ point are fixed.

$$a_{mK} = \frac{GM}{d^2} \tag{1.11}$$

and as a consequence, the mass m, always in the reference frame anchored to M, will have the total acceleration

$$a_{mM} = \frac{GM}{d^2}\left(1 + \frac{m}{M}\right) \tag{1.12}$$

and therefore this observer will see a force equal to

$$F_{mM} = \frac{GMm}{d^2}\left(1 + \frac{m}{M}\right) \tag{1.13}$$

acting on m. It evident that he will assign the speed V_{mM} to the mass m, and, since he will see the force given by (13) acting on this mass, he will write the identity

$$F_{mM} = \frac{mV_{mM}^2}{d} = \frac{GMm}{d^2}\left(1 + \frac{m}{M}\right) \tag{1.14}$$

from which he will obtain relation (9), again. From this equation it follows even the 3rd Kepler's law, revisited by Newton, that is

$$\frac{4\pi^2 d^3}{T^2} = GM\left(1 + \frac{m}{M}\right). \tag{1.15}$$

Actually, the experimental Kepler's law stated that, for all the planets of the solar system, the following relation should be true

$$\frac{4\pi^2 d^3}{T^2} = GM \tag{1.16}$$

a relation that does not depend on the masses of the planets[12].

We can conclude these simple and short recalls of astronomy by saying that Newton, who rereads and corrects the 3rd Kepler's law by introducing the orbiting masses, at the same time rereads and corrects in the same way even the experimental Galilei's law of falling bodies. In fact from (12) obviously we have

$$a_{mM} = \frac{GM}{d^2}\left(1 + \frac{m}{M}\right) = \frac{GM}{R^2}\left(1 + \frac{m}{M}\right) = g\left(1 + \frac{m}{M}\right) \tag{1.17}$$

from which we clearly see that Galilei's law is recovered only when the mass of the falling body is completely negligible in comparison with the mass of the Earth. Let us note[13] that, since the mass of the Earth is equal to 5,976 x 10^{27} grams, for a 1 ton weight[14] the ratio

[12] Let us observe that relation (15) is used every day by the astronomers.

[13] In the processes of orogenesis, where the so called tectonic plates that form the Earth's crust are studied, the results are already different.

[14] Even the notion of weight has to be modified. In fact, let us consider a body A composed by the union of one billion smaller bodies B. It is evident, by (17), that the body A will have a weight one billion larger than

m/M = 1.67 x 10^{-25} ! It is also evident that the formula that gives the period of a pendulum becomes

$$T = 2\pi\sqrt{\frac{l}{g\left(1+\frac{m}{M}\right)}} \tag{1.18}$$

and thus, given the extremely small ratio m/M, at least in terrestrial experiments, we can understand the absolute ineffectiveness of the experiments made by Galilei, Newton and of all the other experiments that are done today in physics laboratories, with the currently available instruments[15]. And it is really disconcerting that the unsurpassable Newton (who, in the two-body problem, corrects the 3rd Kepler's law - which actually is the law of falling bodies in a huge context) does not realize that his correction contemporaneously also applies to Galilei's law (a mistake that concerns us all as well as Newton). In fact, about this subject, he says [1, Proposition VI. Theorem VI]:

The fall of all bodies on the Earth (taking into account the distinct delay arising from the very small resistance of the air) occurs in equal times, as others previously observed[16]; and it is possible to note with great accuracy the equality of these times in pendulums. I have carried out experiments with pendulums made of gold, silver, lead, etc..

Further on, he says:

Let us imagine, in fact, that these terrestrial bodies would be raised until the orbit of the Moon, and together with the Moon, deprived of every movement, would be let free so that they fall in the same time (altogether) onto the Earth; then, for the previous reasons, it is certain that, together with the Moon, they will run equal spaces in equal times.

...And for the same reasoning, the planets that revolve around the Sun[17], let free at equal distances from the Sun, will run, during their fall towards the Sun, equal spaces in equal times. (Even the planets whose masses are contained in his revisited 3rd Kepler's law).

In Proposition X. Theorem X, still on this subject, he states (Newton's tube) *...the bodies fall inside the tube absolutely free and without any appreciable resistance; a piece of gold and a very light feather, let free together, fall with equal speeds, and even if they run a distance of four, six and even eight feet, fall simultaneously on the bottom, as we see in the experiment.*

3. The fundamental role of the orbiting mass and the erroneous calculation of Mercury's perihelion by Le Verrier and Newcomb

The forecasts of Newton concerning the fall of bodies cannot be confirmed, even with the most modern instruments[18], because of the extremely small value of the m/M ratio in the case of falling bodies. But a completely different situation occurs in the solar system, as we can deduce from Table 1.

---

the body B. In common commercial exchanges this has no relevance, but when the weight of a mountain or of a galaxy has to be evaluated. things are very different.

[15] See the current verifications of the so-called Equivalence Principle between inertial mass and gravitational mass: people still believes that it can be verified once Galilei's law of falling bodies has been verified! None seems to be aware that the identity, that has Newton's thoughts as its major assumption, is valid only if Galilei's law is contradicted by the experiments!

[16] This is the unique, very indirect, reference to Galilei's work done by Newton (due to his character).

[17] And not around the common baricentre.

[18] Maybe some indications could be obtained by means of the continuous monitoring of the movement of a pendulum when the Sun rises or downs (in such a case both the Earth, having mass M, and the mass m of the pendulum are in the gravitational field of the Sun).

Planet	mass m [gr]	m/M ratio	M/m ratio	M/m ratio
		(modern values)	(modern values)	(Le Verrier)
Mercury	3,2850E+26	0,0000001652	6.054.795	3.000.000
Venus	4,8714E+27	0,0000024492	408.302	401.847
Earth	5,9760E+27	0,0000030045	332.831	354.936
Mars	6,4500E+26	0,0000003243	3.083.721	2.680.337
Jupiter	1,8971E+30	0,0009537959	1.048	1.050
Saturn	5,6770E+29	0,0002854198	3.504	3.512
Uranus	8,6700E+28	0,0000435897	22.941	24.000
Neptune	1,0520E+29	0,0000528909	18.907	14.400
Pluto	9,8900E+24	0,0000000050	201.112.235	

Table 1

While in the case of the falling bodies we have a m/M ratio whose value is about 10^{-25}, in the case of Mercury we already reach 10^{-5}, while in the extraordinary case of Jupiter we have a value 10^{-3}. In the following we will see the consequences of these facts. In spite of all that, Laplace [6, Vol. III, Chapter V, p. 172], inventor of the method of variation of constants used for the determination of planetary perturbations by Le Verrier and Newcomb, believes that no appreciable errors are done if the masses of the single planets are neglected in comparison with the mass of the Sun, except for Jupiter and Saturn.

We will show that, unfortunately, this assumption accepted by all the astronomers is false.

Let us consider the case of Mercury. Le Verrier did not know the mass of this planet (since Mercury has no satellites) and thus was forced to evaluate the order of magnitude of its mass by means of indirect analyses. In his work [7] he writes:

Dans plusieurs recherches, j'ai reduit cette masse a 1/3,000,000 (of the mass of the Sun), *en consideration des perturbations qu'elle a fait èprouver a la comète d'Encke, dans son passage au perihelie, en 1838. Mais, suivant M. Encke, la masse de Mercure serait encore plus faivle, et ègale a 1/5,000,000 de la masse du Soleil.* ***Nous concluderons donc seulement que cette masse est fort petite, et qu'elle ne peut avoir aucune influence...***

In the following figure we can see the value that Le Verrier [8] assigns to the mass of Mercury, that is 1/3,000,000 of the mass of the Sun.

Nous adopterons pour les masses m, m', ... des planètes, rapportées à celle du Soleil prise pour unité, les nombres suivants :

Mercure...	$m = \frac{1}{3000000} = 0,000\,000\,333\,333$	$\log m = \bar{7},522\,8787$
Vénus.....	$m' = \frac{1}{401847} = 0,000\,002\,488\,51$	$\log m' = \bar{6},395\,9392$
La Terre...	$m'' = \frac{1}{354936} = 0,000\,002\,817\,41$	$\log m'' = \bar{6},449\,8500$
Mars......	$m''' = \frac{1}{2680337} = 0,000\,000\,373\,087$	$\log m''' = \bar{7},571\,8106$
Jupiter....	$m^{iv} = \frac{1}{1050} = 0,000\,952\,381$	$\log m^{iv} = \bar{4},978\,8107$
Saturne....	$m^{v} = \frac{1}{3512} = 0,000\,284\,738$	$\log m^{v} = \bar{4},454\,4455$
Uranus....	$m^{vi} = \frac{1}{24000} = 0,000\,041\,666\,7$	$\log m^{vi} = \bar{5},619\,7888$
Neptune...	$m^{vii} = \frac{1}{14400} = 0,000\,069\,444\,4$	$\log m^{vii} = \bar{5},841\,6375$.

Il serait superflu de discuter ici les incertitudes que présentent plusieurs de ces nombres. Les masses entrant comme facteurs dans les perturbations, on rétablira aisément, quand on le voudra, les valeurs qui seront indiquées par une discussion plus approfondie des observations. Et quant aux différences que ces incertitudes des masses peuvent introduire dans le calcul des demi-grands axes des orbites, elles sont trop faibles pour avoir aucune influence sensible sur les expressions des perturbations.

8.

Fig. 2

Let us assume that Mercury moves along a circular orbit and let us calculate the positions that it occupies along this orbit (see Fig. 3), assuming first that it has a mass m_1 equal to 1/3,000,000 of the Sun, and then that it has a mass m_2 equal to 1/5,000,000 of the Sun. We can imagine, for comparison purposes, that these two masses moves independently around the Sun along the orbit of Mercury, starting at the same time[19] from the point where the mass m_1 of Fig. 3 is located.

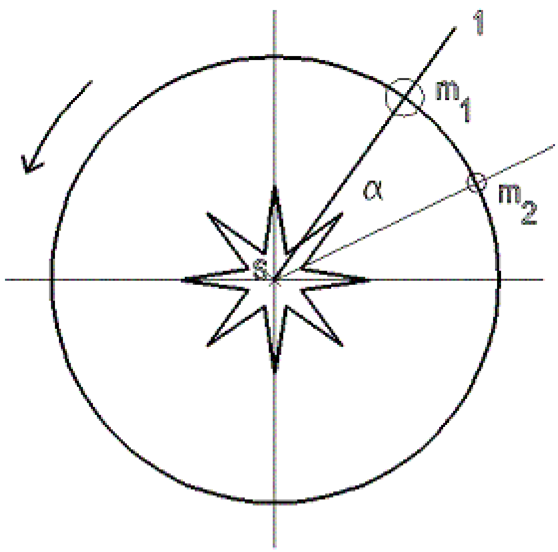

Fig. 3

[19] This is the same as when, on the Earth, we let two masses fall from different heights.

From relation (9) we have two characteristic speeds along this orbit[20], that is

$$V_{m_1} = \sqrt{\frac{6.67\times10^{-8}\times1989\times10^{30}}{57.91\times10^{11}}\left(1+\frac{1}{3.000.000}\right)} = 4.786.340,581\,cm/\sec$$

$$V_{m_2} = \sqrt{\frac{6.67\times10^{-8}\times1989\times10^{30}}{57.91\times10^{11}}\left(1+\frac{1}{5.000.000}\right)} = 4.786.340,262\,cm/\sec\,.$$

Therefore we have a very small speed difference between these two masses which, being equal to 0.3188 cm/sec, we are strongly tempted to set to zero (Laplace). After a complete revolution of the mass m_1, this returns in the start position, while the second has not run the complete orbit, so there is a distance between them. This is what should occur, according to Newton, to two bodies with masses m_1 and m_2 that fall onto the Earth: in this case the acceleration would be variable and the phenomenon would last for a brief time. On the other hand, along a circular orbit the two masses always undergo the same acceleration and *fall forever* onto the Sun. In other terms they revolve around the Sun for an indefinite and monotonically increasing time, which implies that the arc between the masses becomes greater and greater.

Now let us evaluate the length of this arc. For example, after 88 days, the revolution period of Mercury, the arc will be equal to

$$\widehat{m_1 m_2}_{88g} = 0.3188\times88\times24\times3600 = cm\,2.423.900,16\,,$$

and, after a century,

$$\widehat{m_1 m_2}_{100a} = \frac{2.423.900,16}{88}\times36524 = cm\,1.006.028.744\,.$$

The corresponding angle α with the vertex in the centre of the Sun[21] will be

$$\alpha_{rad} = \frac{1.006.028.774}{57,91\times10^{11}} = 0.000174\ ,$$

a value that, expressed in sexagesimal seconds, is equal to

$$\alpha" = \frac{0.000174\times180°}{\pi}\times3600 = 36"\,.$$

Le Verrier, who already had successfully discovered Neptune, being certain of a replica, announced with great resonance to the scientific community of his time that the residual forward shift of the perihelion of Mercury, equal to 38" per century, was not explainable. He attributed the discrepancy to the gravitational effects of a new planet, Volcano, that now we know to be inexistent.

[20] For the values used in these formulas see [9].
[21] The ephemerides of a planet cam be constructed in the same way.

Actually, the previous simple calculations[22] allow us to state that the uncertainty on the value of the mass of Mercury (erroneously considered negligible in the evaluation of planetary perturbations by all the astronomers, according to the above-mentioned opinion of Laplace [6, Chapter V]) would authorize neither Le Verrier to formulate his hypothesis nor Newcomb to express doubts about the validity of Newton's Mechanics, as he did subsequently.

It is important to note that, if we redo the same calculation using the mass of Mercury 1/3,000,000 considered by Le Verrier and the mass known today, which is equal to 1/6,054,795, we have a speed difference equal to 0.40 cm/sec, and thus an angle with the vertex in the centre of the Sun that, after a century, will be equal to

$$0.3188 : 35.6'' = 0.40 : \alpha''$$

from which we have

$$\alpha''_{100} = 44''.$$

Even the work of Simon Newcomb can be criticized in a similar way. He, at the end of his calculations, establishes that the mass of Mercury can at most vary between the limits

$$\frac{1 \pm 0.35}{7900000}$$

or

$$m = [3.4 \times 10^{26}, 1.64 \times 10^{26}][g]$$

If we redo the previous calculations using these values for the mass of Mercury, we find a position uncertainty per century equal to 24" !

However, there is something not convincing in this entire argument, something that concerns the two-body problem like it was formulated by Newton. This is the assumed fixed position of the Sun, in spite of the gravitational action that it undergoes by the planets, and the question concerning the *fall* of planets that move the Sun according to their gravitational mass.

4. Newton and the movement of the Sun

We will see now the procedure that allows us to evaluate the movement of the Sun caused by the planets in the context of Newton's theory; this procedure can be applied to any other gravitational theory.

Let us redo the same calculations done previously: this time we will assume, for the mass of Mercury, the value estimated today m_1 = 1/6,054,795 M, and a value m_2 so small to be practically equal to 0.

In the first case we have

[22] They do not concern directly the theory of planetary perturbations, but the so-called Keplerian orbit that anyway is at the basis of said evaluation. In fact the perturbations concerns the variations of said orbit.

$$V_{m_1} = \sqrt{\frac{6.67\times10^{-8}\times1989\times10^{30}}{57.91\times10^{11}}\left(1+\frac{1}{6.054.795}\right)} = 4.786.340,179\ cm/\sec$$

while, in the second, we find

$$V_{m_2=0} = \sqrt{\frac{6.67\times10^{-8}\times1989\times10^{30}}{57.91\times10^{11}}} = 4.786.339,784\ cm/\sec\ .$$

This implies a speed difference equal to 0,394 cm/sec. Thus in 88 days we have an arc equal to

$$\widehat{m_1m_2}_{88g} = 0.394\times88\times24\times3600 = cm\,2.995.660,8\,.$$

In a century:

$$\widehat{m_1m_2}_{100a} = \frac{2.995.660,8}{88}\times36524 = cm\,1.243.335.398\,.$$

This is equivalent to an angle with the vertex in the centre of the Sun equal to

$$\alpha_{rad} = \frac{1.243.335.398}{57,91\times10^{11}} = 0.000214$$

or

$$\alpha'' = \frac{0.000214\times180°}{\pi}\times3600 = 44''\,.$$

Now it is easy to realize that this is the shift of the Sun towards the γ point caused by the gravitational force of Mercury.

In fact, when we assume that the mass of Mercury is almost equal to zero, this is equivalent to say that the reference frame anchored to the central mass becomes a completely inertial system. If the mass of Mercury (or of a generic planet) is equal to zero, the barycentre K, shown in Fig. 1, coincides with the centre of the Sun and all the observers represented in this figure will be completely equivalent; in other terms the central mass is absolutely at rest (compared with the fixed stars). On the other hand, if we consider the real mass of the orbiting planet, it is evident that the increase of the planet speed (that we have calculated now, and that can be represented by the arc PP' of Fig. 4) has to be attributed only apparently to the secondary mass[23] while it is actually due to the real movement of the central mass in comparison with the fixed stars, which is just equal to the arc SS'=PP'. All these details are shown in Fig. 4.

[23] If the position of the Sun or of the central mass M has to be considered fixed. However it is evident that this locking is impossible; among other things it destroys a huge momentum. In fact we can show that, if V_M and V_m are the speeds of the two masses referred to the baricentre K, the principle of conservation of the momentum, expressed by the relation $M\ V_M = m\ V_m$ = constant, is valid even if it is not satisfied in the non-inertial reference frame anchored to the central mass.

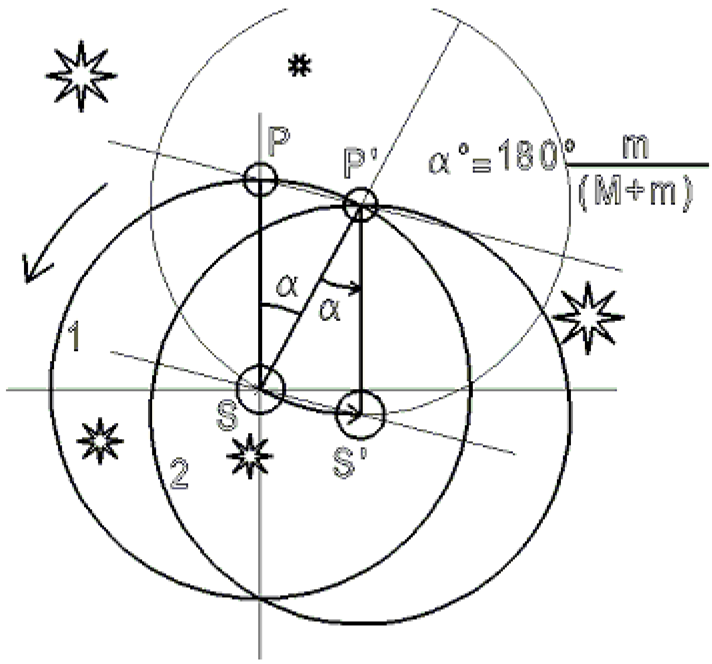

Fig. 4

In fact, let us imagine (see Fig. 4) that the planet, in conformity with Newton's two-body procedure, and thus having its real mass m, moves along the orbit 1 (here we assume, always according to Newton, that the Sun is absolutely at rest). Thus, when the planet returns in position P it moved along its whole orbit. On the other hand, it is clear that, if the planet had a completely negligible mass, once started from P it would reach the position P', if we admit that the arc PP' represents the speed increase of the planet just due to its real mass m. Only in this case the Sun would preserve its original position S [24], a fact that Newton requires anyway. Actually[25], the planet, due to its real mass, will reach the position P', since in the meanwhile the Sun moved going to the position S' (always in comparison with the fixed stars). Thus it seems clear that if we rotate Newton's orbit around the point P', in the same direction of the motion of revolution of the planet, by an angle equal to γ we obtain what occurs compared with the fixed stars.
Thus we can say that (while the orbit 1 is the true orbit when the fixed stars are absent and in a reference frame completely anchored to the central mass - as specified in Note no. 11) the orbit that we obtain, forcing orbit 1 to rotate in the above-mentioned way, is relative to a (heliocentric) reference frame with the origin in the centre of the Sun, with a finite mass M, and that moves compared to the fixed stars.
Then the orbit that in every moment is run by the planet (and that initially coincides only for a small piece with the orbit denoted by 1 in Fig. 4) slowly rotates around the planet, in the direction of the motion of revolution of the planet. The orbit finally reaches position 2 after a whole revolution of the planet. This is shown in Fig. 5.

[24] In the case of Fig. 4, the Sun was originally near a specific star, reported in the figure.
[25] That is, with reference to the fixed stars.

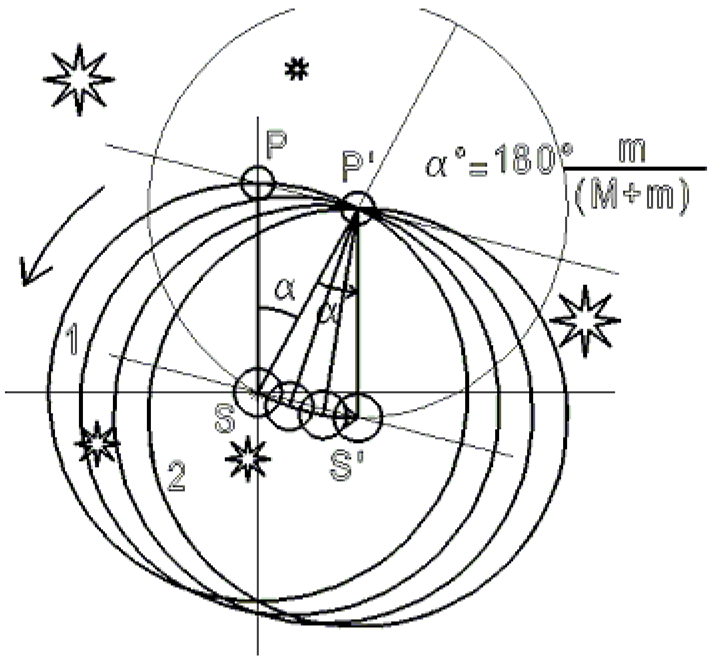

Fig. 5

In case of elliptic orbits, this effect is more evident (see Fig. 6).

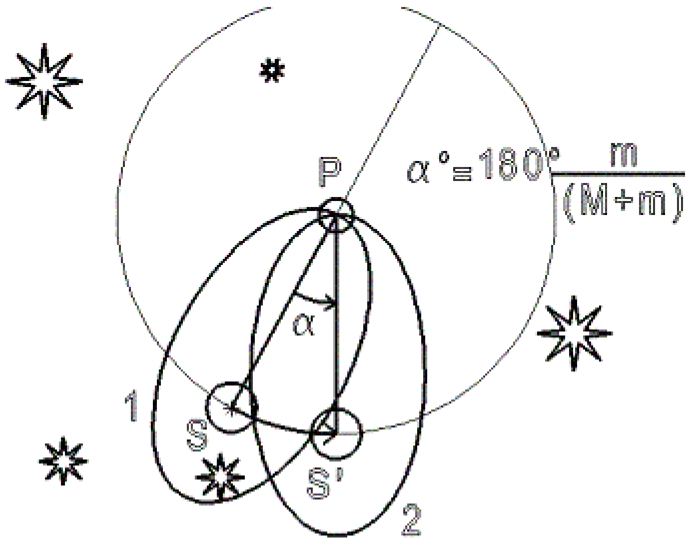

Fig. 6

It is clear that, if the heliocentric observer insists and still considers itself fixed, he will attribute to the planet an apparent forward shift of the perihelion exactly equal to γ, as shown in Fig. 7.

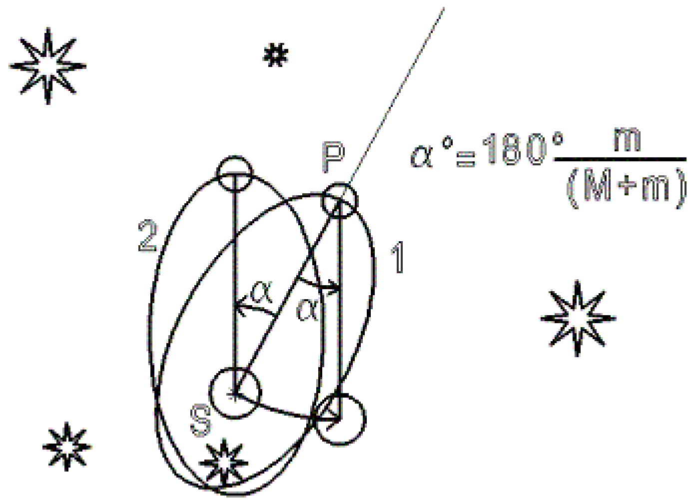

Fig. 7

Now let us calculate theoretically γ. The speed increase of the planet that should be attributed to the real movement of the mass M is given by the difference

$$\Delta V = \sqrt{\frac{GM}{d}\left(1+\frac{m}{M}\right)} - \sqrt{\frac{GM}{d}}\,. \tag{1.1}$$

This difference, multiplied by the revolution period given by (15), gives the arc PP' (Fig. 4) concerning a time equal to the revolution time, that is

$$\widehat{PP'} = \left[\sqrt{\frac{GM}{d}\left(1+\frac{m}{M}\right)} - \sqrt{\frac{GM}{d}}\right] \times \sqrt{\frac{4\pi^2 d^2}{GM\left(1+\frac{m}{M}\right)}} = 2\pi d\left(1-\sqrt{\frac{M}{M+m}}\right) \tag{1.2}$$

or

$$\widehat{PP'} = 2\pi d\left(1-\sqrt{\frac{M}{M+m}}\right) \cong \pi d\,\frac{m}{M+m}\,. \tag{1.3}$$

Dividing such an arc by d, we obtain the angle needed

$$\alpha_{rad} = \frac{\widehat{PP'}}{d} = \pi\frac{m}{M+m} \cong \pi\frac{m}{M} \longrightarrow \alpha° = 180°\frac{m}{M+m} \cong 180°\frac{m}{M}\,. \tag{1.4}$$

5. A simple curiosity

The equation of Newton's gravitation in a reference frame completely anchored to the central mass (see note 11), whose observers see an apparent shift of the perihelion equal to γ, becomes (for the demonstration see [3])

$$F \cong G\frac{Mm}{d^{2+\frac{m}{M}}} \tag{1.1}$$

This equation, in the case of Mercury, becomes

$$F \cong G \frac{Mm}{d^{2,000.000.165}} \tag{1.2}$$

and is practically the same as the equation proposed, at his time, by the astronomer Hall. For more details see [3].

6. The movements of the Sun caused by planets

In the following Table we show the various shifts of the Sun towards the γ point caused by the gravitational forces of the planets acting on the Sun, evaluated by means of relation (21).

	Distance (cm)	**Mass (gr)**	**Revolution (days)**	**Angle α (sexagesimal seconds) on a revolution period 180°·3600·$m/(M+m)$**	**Angle α in a century**
Mercury	5,791E+12	3,2850E+26	88,0000	0,1070	**44,42**
Venus	1,082E+13	4,8714E+27	224,7000	1,5871	**257,97**
Earth	1,465E+13	5,9760E+27	365,2400	1,9469	**194,69**
Mars	2,279E+13	6,4500E+26	687,0000	0,2101	**11,17**
Jupiter	7,783E+13	1,8971E+30	4331,7464	617,4708	**5.206,33**
Saturn	1,427E+14	5,6770E+29	8933,7704	184,8993	**755,93**
Uranus	2,870E+14	8,6700E+28	30683,8124	28,2449	**33,62**
Neptune	4,497E+14	1,0520E+29	60191,5520	34,2715	**20,80**
Pluto	5,947E+14	9,8900E+24	90469,9480	0,0032	**0,00**
Sun		1,9890E+33			

Table 2

While the shift relative to Mercury is practically coincident with the *unexplainable* forward shift[26] of Mercury perihelion, the solar movement due to Jupiter is of the same order than the lunisolar precession. Indeed, if we subtract the movement due to the Earth from the movement due to Jupiter, we have:

$$\beta_{Secolo} = 52,0633'' - 1,9469'' = 50,12''$$

that is practically equal to the value of the actual solar precession. Let us study in more detail this grand phenomenon and its current explanation.

7. Precession and movement of the Sun

We will show the precession phenomenon with the help of the following figure.

[26] On the basis of the previous results, this should be an apparent forward shift due to the real movement of the Sun.

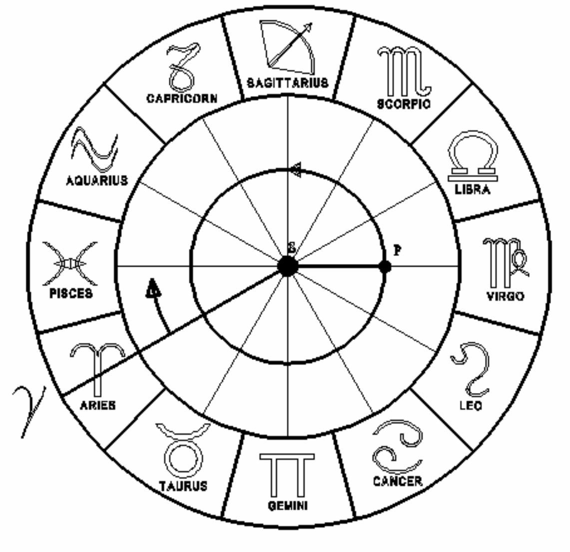

Fig. 8

The Sun is at the centre of the figure, and the orbit of a generic planet, that revolve counterclockwise around it, is shown. Actually the plane of the figure is the same as the plane of the orbit, that also is, approximately, the ecliptic plane, i.e. the apparent movement of the Sun around the Earth projected onto the zodiacal constellations[27]. During the period of revolution of the Earth around the Sun, this rises on the stellar background in different constellations. So the Sun projection will fall, for example, first in the Aries constellation, then in the Taurus, in the Gemini, and so on, until everything restarts after an year.

During the period of revolution of the Earth around the Sun, this rises on the stellar background in different constellations. So the Sun projection will fall, for example, first in the Aries constellation, then in the Taurus, in the Gemini, and so on, until everything restarts after an year[28], there is no apparent reason for which the revolution period of our planet around the Sun should not be rigorously constant[29]. Unfortunately, if we measure the revolution time referred to the said starting point, we can note[30] that this period becomes shorter and shorter when the years pass (the reduction in time is approximately equal to 20 minutes). This does not mean that our time contracts: it implies that some points we considered fixed actually are not fixed. In fact a possible explanation is that the γ point is not fixed but moves slowly clockwise towards the Earth, and thus meets with it before the Earth is able to complete its 360° revolution around the Sun. According to the current version, the celestial equator slowly shifts onto the ecliptic, so the γ point moves towards the Earth with a speed of 50" per year. In such a case, the Earth will meet again this point after a run smaller than 360° but equal to 369° - 50" = 359°59'10", so we can calculate the advance time of the Earth when it reaches this point

If the Earth runs 360° in 365,24 days[31], to run 359°59'10" it will spend a slightly smaller time, i.e.[32]

[27] It is there, on the celestial vault, that we see the Sun and all the planets.
[28] The γ point is the intersection of the ecliptic, the apparent orbit of the sun projected on the celestial vault, and the celestial equator, projection of the terrestrial equator on the celestial vault.
[29] Tropical year coincident with the sidereal year.
[30] In such a case we measure the length of the so-called tropical year.
[31] Sidereal year.
[32] Tropical year. This implies the well-known problem of the calendar and the necessity of the leap year.

$$x = \frac{365.24 - 359.9861}{360} = 365.2259 \quad days$$

Thus we have a time difference equal to

$$365.24 - 365.2259 \equiv 20^m \ \ 17{,}5^s \,. \tag{1.1}$$

Actually, every year, an observer on the Earth sees the Sun that returns in the γ point[33] with an advance time given by relation (25).

Thus this is a relative motion involving the Sun and the γ point, and we should establish first if the γ point moves towards the Sun at a speed of 50" per year, or the Sun moves towards the γ point, or an intermediate case occurs! Newton attributes this relative motion only to the γ point, assuming that the Sun is absolutely fixed[34]. He attributes this motion to the gravitational action of the Sun and the Moon on the equatorial bulge of the Earth. This action would induce in our planet the well-known motion of the axis of a top, but the relevant analytic calculations, as we will see soon, are not clear and can be criticized because of their *ad hoc* character.

On the other hand, if we consider Fig. 9 (where we show the orbits run by M and m around K, and also the orbit that m would run around M in the hypothesis that the latter would be at rest compared to the fixed stars), we see that there is an alternative solution, or a cause neglected before, that is still strictly coherent with Newton's universal gravitation theory.

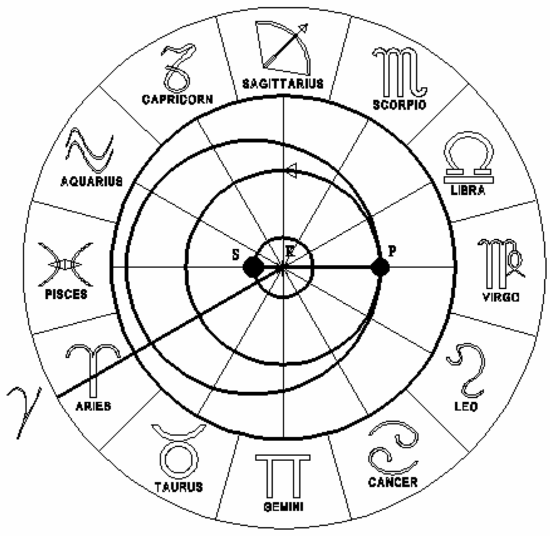

Fig. 9

If the planet P, this time, is identified with Jupiter, it is clear that the centre of the figure will not be occupied by the Sun but will be occupied by the baricentre K of this binary

[33] Even the γ point is not easy to observe astronomically, and is usually considered coincident with the spring equinox.

[34] Here he contradicts himself just another time. In fact in Book III of the Principia (Proposition XII. Theorem XII) he textually states: The motion of the Sun is continuous, but it never goes far away from the common centre of gravity.

system[35]. In the figure we have shown the orbits of the Sun and Jupiter around the baricentre K, and also the orbit of Jupiter around the centre of the Sun. It is quite evident that the Sun, being forced to move around K, will force the orbit of Jupiter (or of another planet) to do the same. On the other hand, if the Sun, for any physical reason, is forced to revolve counterclockwise around a given point, it is clear that a heliocentric observer, unable to sense his own acceleration, will consider himself fixed and will see the zodiacal constellations revolving clockwise around him with the same speed.
See the following Fig. 10 and 11.

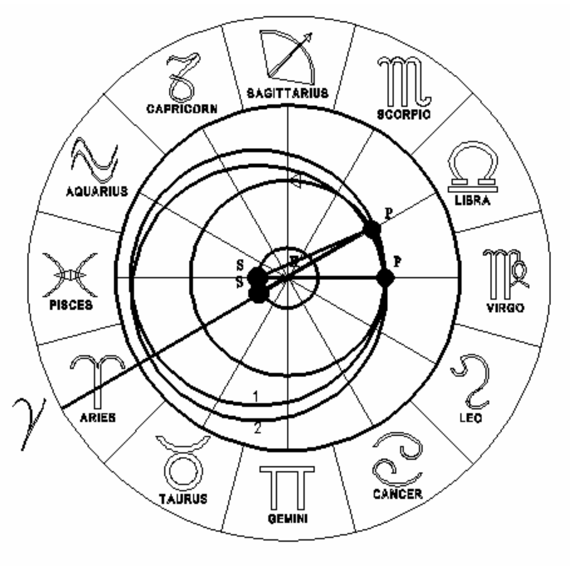

Fig. 10

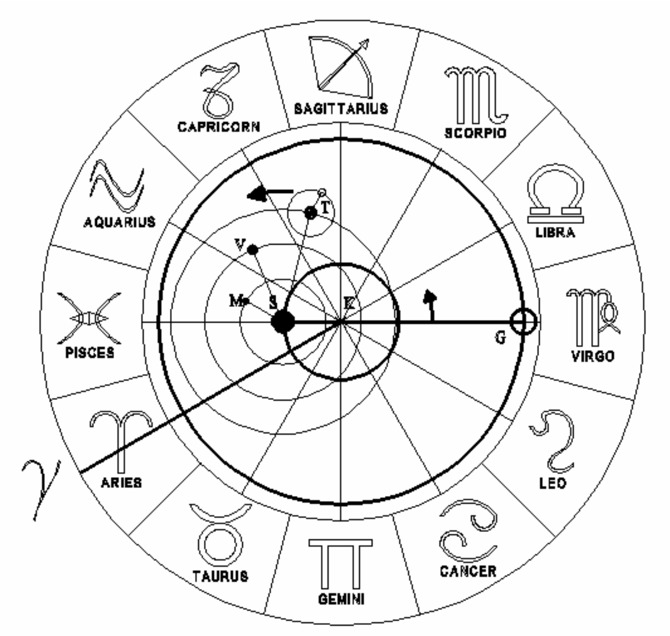

Fig. 11

Of course the results we deduce from these graphical representations can be expressed analytically in a more complex way. This means that the present analytic considerations

[35] We have seen that all the planets cause the same effect, that for each planet is proportional to its mass. Thus it is clear that the effect caused by Jupiter is the greatest. Combining all the various effects, we will have a variable and much smaller average value.

are first order results, that can be refined further by means of more powerful analytical approaches (*n*-body problem).
As we have already observed, if we calculate the apparent speed that Jupiter acquires compared to the Sun because of its gravitational mass, and that actually (due to said movement) should be attributed to the Sun, we have

$$\Delta V = 622.468 \quad cm/\sec .$$

The Sun, pivoting continuously upon Jupiter[36], moves with this speed along a circumference equal to the circumference run by Jupiter itself[37], and this allows us to calculate the time needed by the Sun to run it completely:

$$T = \frac{2\pi d}{\Delta V} = \frac{2\pi 778.34 \times 10^{11}}{622.468} = 7.86 \times 10^{11} \sec \rightarrow 24.900 \text{ years},$$

This value is practically equal to the Platonic year.

Therefore it is evident that both the slight precession of Mercury and the much greater *lunisolar* precession would find, within the scope of Newton's universal gravitation theory, a new and coherent physical interpretation.

A strong internal contradiction would remain in this theory, concerning the current interpretation of the lunisolar precession, but we think that this contradiction, with a deeper study, could only survive in a very reduced aspect.

8. Criticism of the lunisolar precession

The precession of the equinoxes was discovered by Ipparcus [10], who compared the position of Spica (α star in Virgo constellation) in the ecliptic (longitude lambda and latitude β) that he measured in the year 129 B.C. with those previously (144 years) measured by the astronomer Timocharis. As reported by C. Barbieri [10], from this comparison Ipparcus detected a 2° increment in the longitude lambda and no changes in the latitude.

Dividing 2°=7200'' by 144 years, we obtain the rate of change of said longitude, 50'' per year, a value practically equal to the clockwise shift of the Sun towards the γ point, caused mainly by Jupiter.

Now, let us see Newton's justification of this forward shift. First, he makes the implicit hypothesis (absolutely irreconcilable with his own theory) that the Sun is absolutely fixed. We know that it is possible, with his formulas, to calculate the acceleration that the Sun undergoes because of the gravitational forces of the planets. This acceleration was always considered negligible.
Let us use Fig. 12 to understand his explanation.

[36] Considering at present only Jupiter.
[37] Circumference that we could denote "Platonic" circumference.

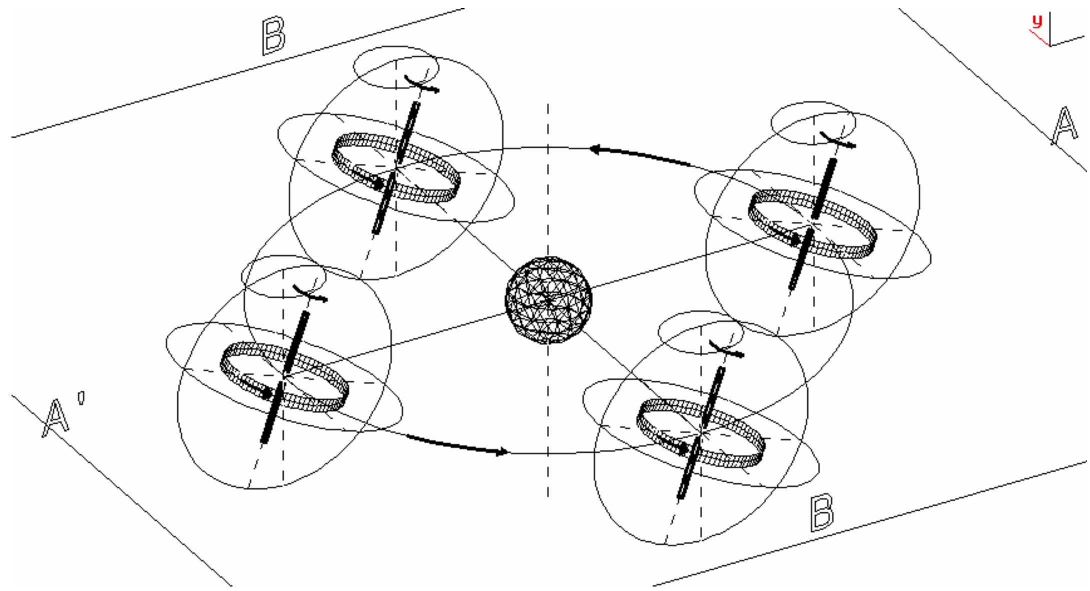

Fig. 12

In this figure we show the orbit run counterclockwise by the Earth around the Sun. The Earth is represented by a handle formed by a flat cylinder (actually a disc) and a rod (terrestrial axis) whose rotation is counterclockwise, too: this only serves to visually enlarge the phenomenon. Actually the equatorial bulge, compared to the terrestrial sphericity, is so small that we should fear instability phenomena for the terrestrial axis. When this handle is at the point A, the part of the disc nearest to the Sun lies above the orbit plane, while the farthest part is below. It is evident that the gravitational action of the Sun, in this position, tends to align the rotation axis orthogonally to the orbit plane. When the Earth moves from A to B, this action reduces until it becomes ineffective at position B. Then it increases again when the Earth moves from B to A'. A similar action is generated by the Moon since it moves around the Earth in a plane different by the equatorial plane.

This action generates a clockwise rotation of the terrestrial axis around an axis perpendicular to the orbit plane (ecliptic). So, a point on the Earth surface has a counterclockwise speed, due to the rotation of the Earth around its axis, and a clockwise speed[38], due to the fact that this axis spans a conical surface when it moves around an axis orthogonal to the ecliptic and passing through the centre of the Earth.

There is no doubt that this physical phenomenon exists, but we must also take into account that the Sun is not at rest, but moves towards the γ point because of the actions of all the planets of the solar system. We can understand what happens only if we evaluate the magnitude of the effects of both the phenomena.

Let us consider in greatest detail the precession theory. Newton, who does not consider other effects besides those due to gyroscopic causes, makes recourse to similitude with the known motions of lunar nodes, and using some questionable assumptions, is able to balance the accounts in a suspect way.

[38] Here we already have a big problem. The so-called sidereal day (passage time of a given star at a specific meridian), that should be a measurement of the rotation speed of the terrestrial axis, evidently already contains the difference of these two speeds. A similar argument holds for the sidereal year.

This approach, used by Newton even in other cases[39] [4], is explicitly criticized *in primis* by Daniel Bernoulli. M. d'Alembert reports, in the introduction of his massive work on the precession [12], that Bernoulli made the same calculations done by Newton, making recourse to less *ad hoc* hypotheses, and found that the so called gyroscopic phenomena can justify only 35" per year, leaving an unjustified discrepancy with the experimental data. D'Alembert [12] reduces the justified value to 30" per year.
This could be seen as a minor problem, but it is not so. In fact we must say that the perihelia of all the planets of the solar system, always referred to said γ point provisionally assumed fixed, move counterclockwise reaching large values in a century, as we can see in the following Table 3.

Planet	Forward shift in sexagesimal seconds per century	Residual forward shift (5000")	Residual forward shift (3500")
Mercury	5.600,82	580,82	2.100,82
Veneus	5.064,42	44,42	1.564,42
Earth	6.190,67	1.170,67	2.690,67
Mars	6.627,20	1.607,20	3.127,20
Jupiter	5.799,66	779,66	2.299,66
Saturn	7.053,27	2.033,27	3.553,27
Uranus	5.344,82	324,82	1.844,82
Neptune	5.129,88	109,88	1.629,88

Table 3

Therefore, even if said gyroscopic phenomena can be used to attribute to the motion of γ (and justify with it) at least 50" per year, that is 5000" per century, we need the theory of planetary perturbations to justify the residue shown in the third column of Table 3, that for Mercury is equal to approximately 580" per century[40]. On the other hand, since the theory of the lunisolar precession allow us to justify only 35" per year, that is 3500" per century, then Newton's theory must justify much greater residues, shown in the fourth column of Table 3. Such a justification is almost impossible unless we introduce a large correction into Newton's Universal Gravitation Theory, and is also beyond the scope of General Relativity.

It is evident that, in case we would be able to explain only 35" with the lunisolar precession, an unprecedented crisis[41] would open for Newton's theory.

Therefore, it is absolutely essential to clarify first, with a kind of casting-out nines, if the criticisms by Bernoulli, Euler and d'Alembert (BED), respectable scientists, have still today their validity, deferring a deeper insight into the whole problem to a subsequent moment.

9. A first verification

We will not report here the complicated analytical treatment of the lunisolar precession [11][12][13]. However it is appropriate to make an extended discussion of its final formula[42] [11, p. 297 and following]:

[39] In the verification of the gravitation law, considering the case of the Earth and the Moon.
[40] Today more precise data are available, but this has no effect on our argument.
[41] Often, crises generally transform into new perspectives.
[42] The solution contains variable terms that are not taken into account, since their average is zero.

$$4.76\times\frac{n^2}{\varphi'}\left[\frac{I_3-I_1}{I_1}\right]\cos\varepsilon \qquad (1.1)$$

where n = 360°, $n/\varphi'=1/366.24$, $\cos\varepsilon=0.9174$[43]. The term enclosed in square brackets contains the moments of inertia of the homogeneous ellipsoid of revolution and represents the index of flattening of the Earth. If it is assumed to be equal to 1/306.8, we have

$$4.76\times\frac{360°}{366.24}\times\left[\frac{1}{306.8}\right]\times0.9174=0.01399°\equiv50.36''. \qquad (1.2)$$

This formula, as already said, putting aside other basic considerations we will do below, contains the ratio of the moments of inertia of the rotation spheroid, whose evaluation is based on the hypothesis that it is homogeneous and has a limited degree of flattening. Instead today we know that the Earth[44] is formed by a thin crust floating on incandescent magma, so a torque applied to the equatorial bulge, rather than apply entirely to the underlying magma by means of friction[45], should and could cause a shift of said crust. This would cause a foreseeable continental drift, the production of specific tectonic faults, the movement of the poles, and should justify the glaciation periods occurred in past epochs (an impossible task for the Newtonian explanation), when almost certainly the rotation axis lied completely in the ecliptic plane. All these considerations also imply that the real ratio between the moments of inertia, which we will denote by the letter β, should be evaluated, in the limit, taking into account only the lenticular solid that we obtain by the previous spheroid when the volume of the entire underlying sphere is subtracted. We can state certainly, given the mentioned structure of the Earth, that the interval of values attainable by β is rather large, and ranges from a value corresponding to a uniform ellipsoid of revolution to the value associated to the said lenticular solid.

It is meaningful that Danjon [11], after having written relation (26), does not calculate[46] the value of the precession but, accepting this value as already known, deduces the unknown value of the ratio between the moments of inertia!

Here, having a dual purpose (to avoid the introduction of totally arbitrary numbers and to give more reliable values), we will make recourse to the free Eulerian nutation of the Earth's axis, just to test the truthfulness or the falseness of the fundamental criticism by BED.

It is well-known that this theory [14,15] is valid when a gyroscope has undergone a brief rotational impulse whose vector was not coincident with the axis of revolution of the ellipsoid. This impulse acted in the past and caused an oscillation of the terrestrial axis that still lasts today[47] even in absence of persistent torques.

In such a case the frequency of said free nutation is a function that depends only on the speed of rotation of the gyroscope axis and the inertia ratio. Therefore we have the known formula [14,15]

[43] Where ε = 23°27' is the inclination angle of the terrestrial axis, and φ' is the time variation of the γ point.
[44] 2/3 of whose surface are covered by water.
[45] Probably creating the terrestrial magnetic field.
[46] This formula already contains the precession value represented by φ'.
[47] Probably caused by the fall of a big meteorite, an impact occurred in past epochs.

$$f = \frac{1}{2\pi}\left[\frac{I_3 - I_1}{I_3}\right]\ \omega_3 = \frac{1}{2\pi}\beta\,\omega_3, \tag{1.3}$$

where ω_3 is the frequency of rotation of the Earth around its axis, equal to 2π/day.
Now, if we insert in this formula the value β calculated with relation (26), assuming that the value of the lunisolar precession is known and is equal to 50.36", as done by Danjon, we have [14, p. 265]

$$f = \frac{1}{2\pi}\left[\frac{I_3 - I_1}{I_1}\right]\omega_3 = \frac{1}{2\pi}(0.00326)(2\pi) = 0.00326 \quad rivol\,/\,giorno$$

i.e. a frequency of 0.00326 revolutions per day, and a related period of approximately 1/0.00326=307 days, in complete contrast with the experimental value that is approximately 430 days [15, Vol. I, p. 179, 180 and 181] [14, p. 257, 265].
This considerable discrepancy between theory and experiment is attributed, only in the specific case of the Eulerian nutation, to the fact that the Earth is not a rigid body etc. etc. [15, Vol. I, p. 179, 180 and 181] [14, p. 257, 265]. However, when this value of the ratio is inserted in relation (27) and the theoretical value coincides with the experimental result, in such a case, to avoid a catastrophic result, everyone forgets completely and strangely that the Earth is not rigid at all !
It is evident that this behaviour, certainly not orthodox, is due exclusively to the fact that all the 50" must be explained with the gyroscopic phenomena, otherwise an unprecedented crisis would open for Celestial Mechanics, as we already said.
On the other hand, if, given the extreme simplicity of relation (28), we assume that the Eulerian precession is known and equal to observed value, then, thanks to (28), we are able to obtain for beta a value free by any criticism. In such a way we have

$$\beta = \frac{I_3 - I_1}{I_1} = \frac{1}{430}\,.$$

Now, if we insert this value into relation (27), we obtain for the precession

$$4.76 \times \frac{360°}{366.24} \times \left[\frac{1}{430}\right] \times 0.9174 = 0.00998° \equiv 35.9" \tag{1.4}$$

.

Thus the criticisms by Bernoulli, Euler and d'Alembert about Newton's explanation are well-founded !!!

Anyway, it is important to note that Bernoulli, Euler and d'Alembert did not criticized the value of the moment of inertia but concentrated their criticism on the ratio between the solar action and the lunar action, chosen *ad hoc*. About this subject we must state precisely that the correct similitude that Newton invokes between the forward shift of the lunar nodes, due to the action of the Sun, and the gyroscopic phenomena is certainly undoubted and undeniable. However, the magnitude of the forward shift of the lunar nodes, since even the Earth is, very strongly, moved by the Moon, cannot be due entirely to the action of the Sun[48]. This implies, evidently, a conspicuous reduction of the action of the Sun and, by consequence, of the Moon, but we will not take into account this effect, at least now.

Having said the above, even only on the basis of the criticism by BED, a necessity arises[49] we should justify, with the theory of planetary perturbations, a difference that, on the basis of these first determinations, is approximately 15" per year, that is 1500" per century, values that are, as previously mentioned (see Tab. 3) completely unjustifiable according to the most modern gravitation theories.

A suitable route for a possible solution of this big and never mentioned problem could be the following. (i) Let us admit temporaneously the validity of the criticism by d'Alembert, according to which the gyroscopic effects can justify approximately 30". (ii) Let us neglect temporaneously (this is to the advantage of the gyroscopic thesis) the strong reduction of the lunar nodes caused by the movement induced. (iii) Let us note that the correction of the ratio between the moments of inertia, neglected by d'Alembert, implies another reduction of 72%. Then, with (i), (ii), (iii) we can say, in a first instance, that the gyroscopic effects can justify only 15"~ 20".

The movement induced by Jupiter, taking into account the braking action by Earth, Saturn and Uranus, could justify 52-2-6.3-0.34=43", and thus we would have 43" that, added to those due to gyroscopic effects, would give a precession value very near the experimental value 50". So, finally, the relative displacement between the γ point and the Sun could be attributed partially to the undeniable motion of the Sun and partially to the gyroscopic effects, and this would save the Newtonian law of the inverse square of the distance. And all of this would be the greatest success of the unsurpassable Newton.

Now that we have seen the possibility of this solution, we must also observe that relation (27) is used by inserting the sidereal year in it: the sidereal year should represent exactly the revolution time of the Earth around the Sun. It is measured expecting the re-alignment of the Sun and the Earth with a specified fixed star, that is with the famous γ[50]. It is clear that this would be true only if the Sun would be absolutely at rest, which is absolutely unconceivable!

Thus we find another very strong and very evident conclusion: if the Sun moves for any cause of a specified quantity towards the γ point, the sidereal year ceases to be coincident, as today is assumed, with the revolution period of the Earth around the Sun!

This is approximately the same fact rightly and inveterately stated even by the **Binary Research Institute** group of Walter Cruttenden [15], who attributes to a star[51] binary companion of the Sun, and not mostly to Jupiter, the entire phenomenon of the lunisolar precession.

[48] This is a subject that, for lack of available space, cannot be faced here.
[49] Actually, this crisis had to emerge already much time ago, and it should be instructive to understand why this did not happen.
[50] Thus this point either is fixed or not. Anyway, its motion is not easy to calculate, since we must make recourse to the calculation of the spring equinox, which in turn is related to the motion of the Sun.
[51] A star that still should be discovered.

10. Conclusions

In this paper we proposed to outline a problem, hoping that it was sufficiently sketched, and to suggest, albeit summarily, a possible solution that, however, requires a further analysis of many disparate factors, among which let us mention even the finite speed of the gravitational action.

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