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Physic and Seismic Engineering

Critics to Vibration Mechanics

A possible re-reading of Planck's equation

Gravitational Waves Quantization

First Part

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The ***Truth****, by defining it, should be the easiest and innocent* ***Lie****. (C. S.)*

Abstract

If Ω is the forcer pulsation and ω the one of the oscillating system, it's known that, by the actual Vibrations' Mechanics, we have a unique resonance condition, characterized by the identity

$$\Omega = \omega ,$$

in the hypothesis of null damping.

It's possible to show the previous identity is only one of the infinite and numerable resonance conditions that can verify in the experimental reality and that the most general condition is given instead by the relation

$$\omega = \Omega n \qquad (n = 1, 2, 3...)$$

in which n is any natural entire number.

In this issue we'll examine the immediate implications this involves in seismic engineering and in theoretical physics.

1. Introduction

One of the most important physic phenomenas through which any kind of energy can pass from a system to another one, is the so-called resonance phenomenon. So, for example, the light, an eminently oscillating phenomenon, can be absorbed from matter or cross through it undisturbed, either a seismic wave can dangerously shake a structure until producing its collapse, or leaving it completely undamaged. It's verified the one or the other of the said situations apiece the system receiving the action goes or not in resonance by the external cause that acts on it.
From that the fundamental importance to deepen study the said phenomenon.
About it, it's poised, radicated and general conviction that it only verifies when forcer resonance Ω coincides with ω of the system in survey, without internal attrition, we have a unique condition

$$\omega = \Omega . \tag{1.1}$$

Instead it's very easy and immediate to see that the previous relation is only one of the infinite and *numerable* resonance conditions available in the daily and macroscopic reality.

Infact we can show, both in a very intuitive way and analytically, that, generally, is valid the relation

$$\omega = \Omega n \qquad (1, 2, 3 ...) \tag{1.2}$$

where n must be rigorously an entire number.

Let's consider various oscillating systems that have a single degree of freedom (Single Degree Of Freedom (SDOF)). The extension to continuate systems, discussed later, is immediate. Fig. 1 shows a pendulum, deprived of attrition, whose period is give by the known relation (1.3)

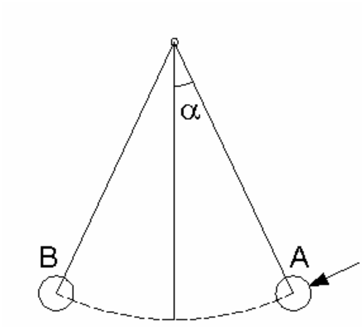


fig. 1

$$\tau = 2\pi \sqrt{\frac{l}{g}} . \tag{1.3}$$

Fig. 2 shows the case of a buoyant to whom is hanging a mass m. if the said mass oscillates around the equilibrium position, the period is given by the relation

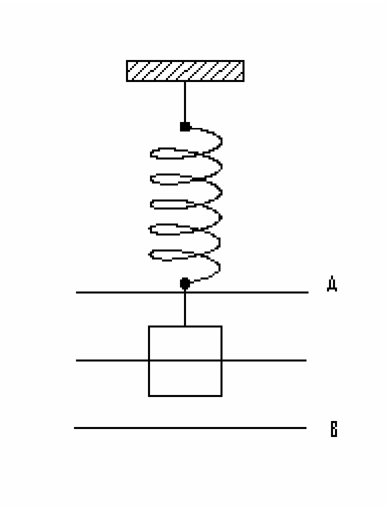


fig. 2

$$\tau = 2\pi\sqrt{\frac{m}{k}} \qquad (1.4)$$

where m is the mass and k is the elastic constant of the spring.

Fig. 3 show the case of a portal with an infinitely rigid transverse (deformed by Grinter[1]) for which is still valid the (1.4) and where

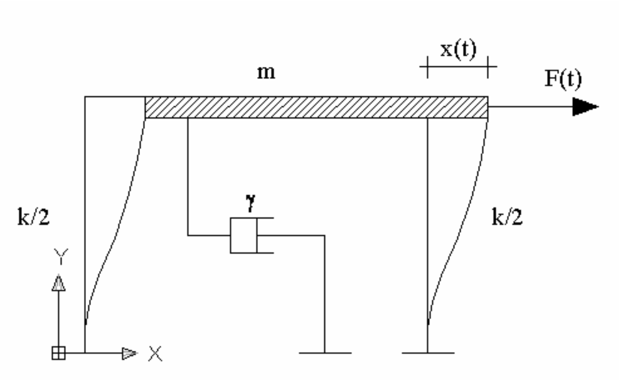


fig. 3

$$k = 2\frac{12EI}{h^3}, \qquad (1.5)$$

E is the elasticity module of the arrects, I the moment of inertia of those ones in the direction of the movement and h their height.

[1] By this hypothesis, for pure semplicity, we avoid the movement calculum of the casement casued the transverse deformabletely, hypothesis that can anyway be removed.

Maybe it's opportune the following circumstantiation. As we can see from (1.4), systems' period now considered is independent by the initial elongation of the oscillator. Fig. 4 shows the generic scheme of the said harmonic oscillator.

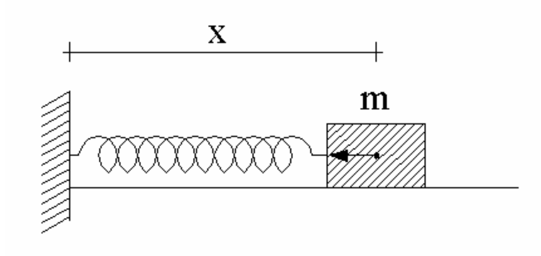


fig. 4

In that case the recall force acting on the mass is directly proportional to the movement and so it's given by the equation

$$F = -k\,x\,. \tag{1.6}$$

So the mass is subjected to a recall force that the more is stronger the more is distant the blastoff point from the origin. You should attempting to think that the period τ of that oscillator would be the more littlest the more bigger is the initial elongation of the spring and this oppositely to what we deduce from (1.4). But we need to observe that if it's true that the force is the more bigger the more bigger the elongation, it's also true that, to a big initial elongation, also corresponds a bigger space the mass has got to run. Then, although the mass goes to be subjected to a bigger medium velocity, it will anyway run a bigger space, from that we have the independence of the period from the elongation and the validity of (1.4).

It's not the same in the case of the pendulum of the fig. 1. The direct proportionality between the recall force and the blastoff arched, corresponding to the angle α, is valid until it's licit to confound α (expressed in radiants) by the value of $sen\,\alpha$ and so when

$$\sin\alpha \cong \alpha\,.$$

When this happens we fall back in the famous isochronism of the little oscillations (Galilei) so we can affirm that the period of the pendulum is still independent from the initial elongation. The analysis that follows will only consider harmonic oscillators and so the linear ones. This involves that by considering, in this case, only oscillators whose recall force follows Hooke's law, by paraphrasing Galilei, we can say that for them is always valid the isochronism of the oscillations, even with a short or a large initial or intermediate elongation.

After said this, now we want to intuitively take count of the validity of (1.2), and after to go on by the analytical way.

The mass m, suspended in A, take the time τ to describe the path ABA, the same will happen for the other systems represented in the previous figures. If, at the moment the said mass reoccupies the position A, we apply a force for a little time, as it's visible in fig. 1, and we continually repeat everytime the said mass is in A, we provoke the so-called resonance phenomenon so pendulum oscillation amplenesses will increase indefinitely.

So if we denote by T the period with we apply the said impulses we have the obvious identity

$$T = \tau \tag{1.7}$$

that, in pulsation terms, can also be written in the form

$$\frac{2\pi}{T} = \frac{2\pi}{\tau} = \Omega = \omega \, , \tag{1.8}$$

from this we have the classic and unique resonance condition. But it's also evident we're not constricted at all to apply the said impulses everytime the mass reoccupies *consecutively* the position A to provoke the unlimited increment of the oscillation ampleness. Infact if, with opportune and variable pauses, we apply the said impulses everytime is true the identity

$$T = \tau \, n \qquad (n = 1, 2, 3 \ldots) \tag{1.9}$$

where n must be obviously and rigorously an entire number, even in this more general case we are able to produce resonance phenomenon. Infact only if n is a an entire number the action provoking our impulse will always agree or in assonance with the natural move of the oscillating mass so our action will never be able to provoke an undesired stopping of the mass movement, thing that surly would happen if the said numbers weren't entire or, at the maximum, fractional ones. From (1.9) we'll have evidently the identity[2]

$$\frac{2\pi}{T} = \frac{2\pi}{\tau \, n} \Rightarrow \Omega = \frac{\omega}{n} \Rightarrow \omega = \Omega \, n \, . \tag{1.10}$$

But the notice give by (1.10) only describes in very empirical way and only in faint lines the said phenomenon, neither by it we can understand the important role of the attrition force, and other necessary to describe the entire dynamics of the phenomenon. With the (1.10), of this fundamental phenomenon we're only able to see, and out of focus, the extreme peak of an iceberg, while we're also escaping the punctual description of the big deafened *part* . Instead analytic solutions that follow will allow us to calculate *punctual* movements, accelerations and velocity of the mass subjected to vibrations and so to certainly establish the so-called answer of the oscillator.

[2] The motion attached to this file show this example.

2. Analytic evolutions

In order to study the concepts expressed previously let's build a forcer to let allow us to apply an impulse, of length Δt, with a certain pulsation Ω. By a development in series of Fourier we can consider the forcer

$$F = m\delta\left\{\frac{a}{\pi}-\frac{2}{\pi}\left[\frac{1}{1}\sin(1a)\cos(\Omega t)-\frac{1}{2}\sin(2a)\cos(2\Omega t)+\frac{1}{3}\sin(3a)\cos(3\Omega t)-...\right]\right\}, \quad (2.1)$$

where m is the oscillating mass, δ its acceleration and $(2a)$ represents the application break of the same one $\left(\Delta t=\frac{2a}{\Omega}\right)$. The said forcer, by gradually considering most terms of the series, tends to a rectangular impulse (see fig. 3).

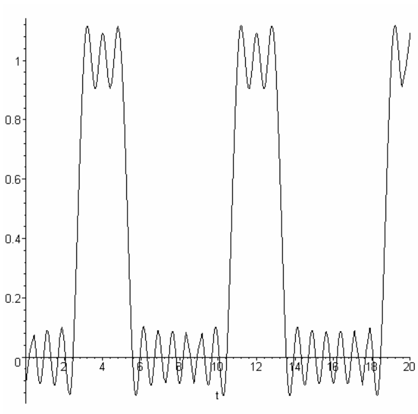


fig. 3

In the case of an oscillator deprived of attrition we have the equation

$$\frac{d^2x}{dt^2}+\omega^2 x = \delta\left\{\frac{a}{\pi}-\frac{2}{\pi}\left[\frac{1}{1}\sin(1a)\cos(\Omega t)-\frac{1}{2}\sin(2a)\cos(2\Omega t)+\frac{1}{3}\sin(3a)\cos(3\Omega t)-...\right]\right\}. \quad (2.2)$$

Because of its linearity we can solve it by summing the various solutions of the following equations

$$\begin{array}{llll}\ddot{x} & +\omega^2 x & = & \frac{\delta a}{\pi} \\ \ddot{x} & +\omega^2 x & = & -\frac{1}{1}\frac{2\delta}{\pi}\sin(1a)\cos(1\Omega t) \\ \ddot{x} & +\omega^2 x & = & +\frac{1}{2}\frac{2\delta}{\pi}\sin(2a)\cos(2\Omega t) \\ \ddot{x} & +\omega^2 x & = & -\frac{1}{3}\frac{2\delta}{\pi}\sin(3a)\cos(3\Omega t) \\ . & . & . & . \end{array} \quad (2.21)$$

The first equation has the solution

$$x_1(t) = C_{11}\cos(\omega t) + C_{12}\sin(\omega t) + \frac{\delta a}{\pi\omega^2}. \tag{2.22}$$

The second one, posed

$$\Xi = -\frac{2\delta}{\pi}\sin(a), \tag{2.23}$$

has the solution

$$x_2(t) = C_{21}\cos(\omega t) + C_{22}\sin(\omega t) + \Xi\frac{\cos(\Omega t)}{\omega^2 - \Omega^2}, \tag{2.24}$$

the third one, posed

$$\Upsilon = \frac{1}{2}\frac{2\delta}{\pi}\sin(2a), \tag{2.25}$$

$$x_3(t) = C_{31}\cos(\omega t) + C_{32}\sin(\omega t) + \Upsilon\frac{\cos(2\Omega t)}{\omega^2 - 2^2\Omega^2} \tag{2.26}$$

end the forth one, posed

$$\Gamma = -\frac{1}{3}\frac{2\delta}{\pi}\sin(3a), \tag{2.27}$$

is

$$x_4(t) = C_{41}\cos(\omega t) + C_{42}\sin(\omega t) + \Gamma\frac{\cos(3\Omega t)}{\omega^2 - 3^2\Omega^2} \tag{2.28}$$

so the general solution is

$$x(t) = A\cos(\omega t) + B\sin(\omega t) + \frac{\delta a}{\pi\omega^2} + \Xi\frac{\cos(1\Omega t)}{\omega^2 - 1^2\Omega^2} + \Upsilon\frac{\cos(2\Omega t)}{\omega^2 - 2^2\Omega^2} + \Gamma\frac{\cos(3\Omega t)}{\omega^2 - 3^2\Omega^2} + .. + \tag{2.3}$$

The first and the second term of the previous solution (and so the two terms characterized by the constants A and B to be determined by imposing the initial conditions) take to the note sinusoidal solution, to which we sum the effects of the other terms of the second member. If we study the function constituted by these last ones we have the following graphs (fig. 4 and 5). On the axis n is reported the relation Ω/ω, on the axis t the time and on the vertical axis the answer x(t) of the oscillator.

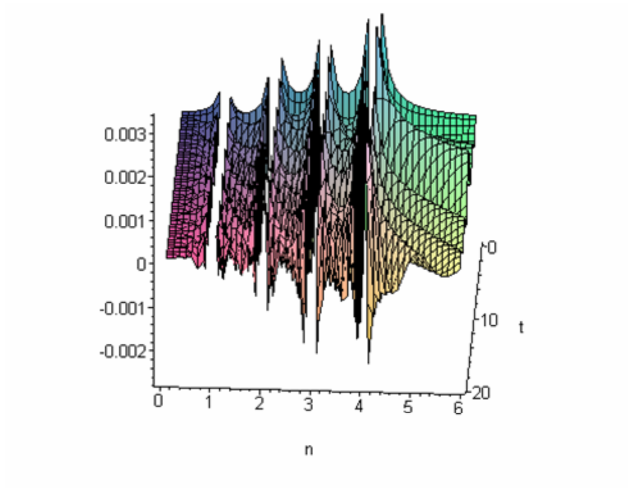


fig. 4

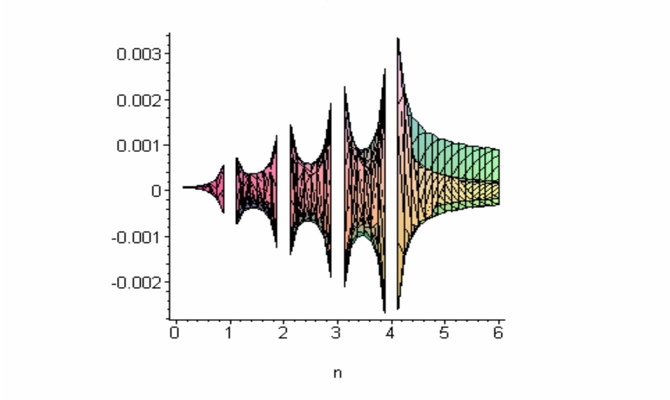


fig. 5

The fig. 5 shows frontally the graph of the fig. 4. From the solution (2.3) and the said figures we deduce we have more resonance conditions when the relation Ω / ω is equal to an entire number. Infact the various denominators of the second member of (2.3) annul when

$$\omega = \Omega n \qquad (n = \pm 1, \pm 2, \pm 3 \ldots) \tag{2.4}$$

Because the generic term of the solution, that is synthetically expressible by the formula

$$\sum_{n=1}^{n=\infty} \left\{ (-1)^n \frac{2\delta}{\pi n} \sin(na) \right\} \frac{\cos(n\Omega t)}{\omega^2 - n^2\Omega^2} = \sum_{n=1}^{n=\infty} \mathrm{T}_n \frac{\cos(n\Omega t)}{\omega^2 - n^2\Omega^2}, \tag{2.5}$$

Accedes indefinite values when is verified the (2.4).

Clearly, by the study of the equation (2.2), even if is fully reconfirmed the intuition of the resonance condition (1.10) and the simple experiment shown in the attached motion, we do not arrive to big results. The mass position remains undetermined just in proximity of the resonances.

This envolves a complete indetermination of the elongation, of the velocity and of the acceleration of the rounds to which is subjected the oscillating system. If we don't want to recur to a renormalization[3], It's necessary not to neglect the attrition force.

[3] Even the couple proton-electron constitutes an oscillating system. And even for this system are valid these *considerations. In physics, without knowing the punctual precision around the nucleus, we impose that the probability to find the said particle in teh* ientire and infinite space surrounding the atom is equal to the unit!

3. The answer of the damping oscillator

In that case we have the equation

$$\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+\omega^2 x=\delta\left\{\frac{a}{\pi}-\frac{2}{\pi}\left[\frac{1}{1}\sin(1a)\cos(1\Omega t)-\frac{1}{2}\sin(2a)\cos(2\Omega t)+\frac{1}{3}\sin(3a)\cos(3\Omega t)+..\right]\right\}. \quad (3.1)$$

We proceed in the same way of the previous case. We have the following example of equations

$$\begin{array}{llllc} \ddot{x} & +\gamma\dot{x} & +\omega^2 x & = & \frac{\delta a}{\pi} \\ \ddot{x} & +\gamma\dot{x} & +\omega^2 x & = & -\frac{1}{1}\frac{2\delta}{\pi}\sin(1a)\cos(1\Omega t) \\ \ddot{x} & +\gamma\dot{x} & +\omega^2 x & = & +\frac{1}{2}\frac{2\delta}{\pi}\sin(2a)\cos(2\Omega t) \\ \ddot{x} & +\gamma\dot{x} & +\omega^2 x & = & -\frac{1}{3}\frac{2\delta}{\pi}\sin(3a)\cos(3\Omega t) \\ . & . & . & . & . \end{array} \quad (3.11)$$

The first equation has the solution

$$x_0(t)=C_1\,\mathrm{e}^{\left[\frac{1}{2}\left(-\gamma+\sqrt{\gamma^2-4\omega^2}\right)t\right]}+C_2\,\mathrm{e}^{\left[\frac{1}{2}\left(-\gamma+\sqrt{\gamma^2-4\omega^2}\right)t\right]}+\frac{\delta a}{\pi\omega^2}. \quad (3.12)$$

The second one, posed

$$\mathrm{T}_1=-\frac{1}{1}\frac{2\delta}{\pi}\sin(1a) \quad (3.13)$$

has the solution

$$x_1(t)=C_1\,\mathrm{e}^{\left[\frac{1}{2}\left(-\gamma+\sqrt{\gamma^2-4\omega^2}\right)t\right]}+C_2\,\mathrm{e}^{\left[\frac{1}{2}\left(-\gamma-\sqrt{\gamma^2-4\omega^2}\right)t\right]}+\frac{\mathrm{T}_1\left[\left(\omega^2-\Omega^2\right)\cos(\Omega t)+\gamma\,\Omega\sin(\Omega t)\right]}{\left(\Omega^2-\omega^2\right)^2+\gamma^2\Omega^2}. \quad (3.14)$$

This equation, as it's known, can be written in the form

$$x_1(t)=C_1\,\mathrm{e}^{\left[\frac{1}{2}\left(-\gamma+\sqrt{\gamma^2-4\omega^2}\right)t\right]}+C_2\,\mathrm{e}^{\left[\frac{1}{2}\left(-\gamma-\sqrt{\gamma^2-4\omega^2}\right)t\right]}+\frac{\mathrm{T}_1\cos\left(\Omega t-\Theta_1\right)}{\sqrt{\left(\Omega^2-\omega^2\right)^2+\gamma^2\Omega^2}} \quad (3.15)$$

where

$$\Theta_1=\arctan\left(\frac{\gamma\,\Omega}{\Omega^2-\omega^2}\right). \quad (3.16)$$

The third (or umpteenth equation), posed

$$\mathrm{T}_2 = \frac{1}{2}\frac{2\delta}{\pi}\sin(2a) \tag{3.17}$$

has the analog solution

$$x_n(t) = C_1\,\mathrm{e}^{\left[\frac{1}{2}\left(-\gamma+\sqrt{\gamma^2-4\omega^2}\right)t\right]} + C_2\,\mathrm{e}^{\left[\frac{1}{2}\left(-\gamma-\sqrt{\gamma^2-4\omega^2}\right)t\right]} + \frac{\mathrm{T}_n\cos(n\Omega t - \Theta_n)}{\sqrt{\left(n^2\Omega^2-\omega^2\right)^2 + n^2\Omega^2\gamma^2}} \tag{3.18}$$

with the positions

$$\mathrm{T}_n = (-1)^n \frac{1}{n}\frac{2\delta}{\pi}\sin(na) \quad e \quad \Theta_n = \arctan\left(\frac{n\Omega\gamma}{n^2\Omega^2-\omega^2}\right). \tag{3.19}$$

So the final solution results to be this

$$x(t) = A\mathrm{e}^{\left[\frac{1}{2}\left(-\gamma+\sqrt{\gamma^2-4\omega^2}\right)t\right]} + B\mathrm{e}^{\left[\frac{1}{2}\left(-\gamma-\sqrt{\gamma^2-4\omega^2}\right)t\right]} + \frac{\delta a}{\pi\omega^2} + \sum_{n=1}^{n=\infty}\frac{\mathrm{T}_n\cos(n\Omega t - \Theta_n)}{\sqrt{\left(n^2\Omega^2-\omega^2\right)^2 + n^2\Omega^2\gamma^2}}. \tag{3.2}$$

Said solution is composed by a transient[4] part (terms affected by the constants A e B) and by a stationary part.

The last one is graphically represented by the figures 6 and 7.

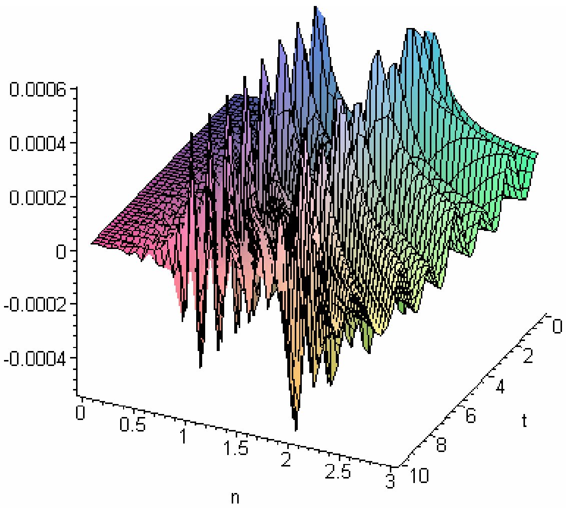


fig. 6

[4] Effectually, after a certain time, these terms tend to zero. What remains in a persistent way is the stationary pehnomenon represented by the remaining part of the equation (3.2).

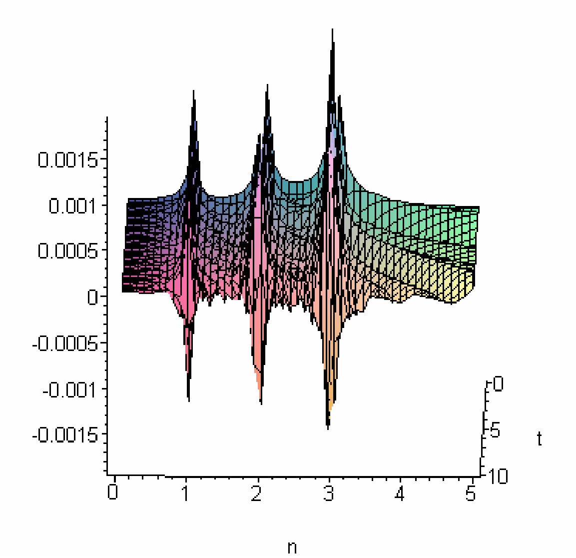


fig. 7

If we consider the solution of the known equation

$$\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+\omega^2 x=\delta\cos(\Omega t) \tag{3.3}$$

end then

$$x(t)=\_C_1\,e^{\left[\left(-\frac{\gamma}{2}-\frac{\sqrt{\gamma^2-4\omega^2}}{2}\right)t\right]}+\_C_2\,e^{\left[\left(-\frac{\gamma}{2}+\frac{\sqrt{\gamma^2-4\omega^2}}{2}\right)t\right]}+\frac{\delta\cos(\Omega t-\Theta)}{\sqrt{\left(\Omega^2-\omega^2\right)^2+\Omega^2\gamma^2}} \tag{3.4}$$

we see how the (3.2) would be an obvious generalization of the (3.4). to underline the differences towards the said solutions it's necessary a numeric confront.

4. The confront

We'll pose in comparison the answers of an identical damping harmonic oscillator, once it's subjected to a sinusoidal forcer

$$F = m\,\delta\cos(\Omega t) \tag{4.1}$$

and once subjected to an impulsive forcer

$$F = \frac{4}{\pi} m\,\delta\left[\frac{1}{1}\sin(1a)\cos(1\Omega t) + \frac{1}{3}\sin(3a)\cos(3\Omega t) + \frac{1}{5}\sin(5a)\cos(5\Omega t) + .. + \right] \tag{4.2}$$

both of them represented in fig. 8.

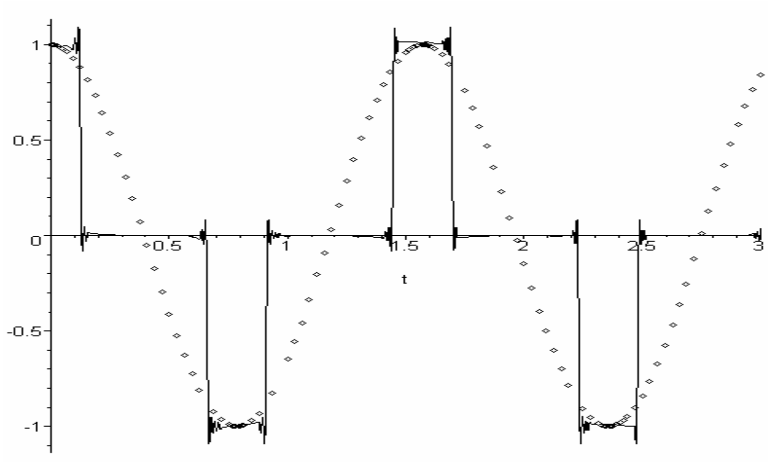


fig. 8

In this last case we have the following equation

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega^2 x = \frac{4}{\pi}\delta\left\{\left[\frac{1}{1}\sin(a)\cos(\Omega t) + \frac{1}{3}\sin(3a)\cos(3\Omega t) + \frac{1}{5}\sin(5a)\cos(5\Omega t)..\right]\right\}. \tag{4.3}$$

Posed

$$\mathrm{T}_n = \frac{4\delta}{\pi n}\sin(n\,a) \qquad (n = 1, 3, 5, 7, ..), \tag{4.31}$$

we have the following stationary solution

$$x(t) = \mathrm{T}_n \frac{\cos(n\Omega t - \Theta_n)}{\sqrt{\left(n^2\Omega^2 - \omega^2\right)^2 + n^2\gamma^2\Omega^2}} \qquad (n = 1, 3, 5, 7, ...) \tag{4.4}$$

The (4.4) presents resonance peaks[5] everytime

[5] Thy are strongly pronounced when the dissipative term is too low.

$$\frac{\omega}{\Omega} \cong n = (2m-1) \qquad (m = 1, 2, 3..) \tag{4.5}$$

end so when the said relation is equal to an entire uneven number.
Let's study the answer of the stationary part of the (4.4) and let's assume the following values

$$\Omega = 4, \quad a = 0.5, \quad \delta = 10, \quad g = \gamma = 0.01. \tag{4.6}$$

In that case the period with the impulses are applied is $T = 2\pi / \Omega = 2\pi / 4 = 1.57$ sec., while the duration of the impulse is equal to $\Delta t = 2a / \Omega = 0.25$ sec..
Figures n. 9, 10 and 11 let us evaluate the maximum movement of the oscillator.

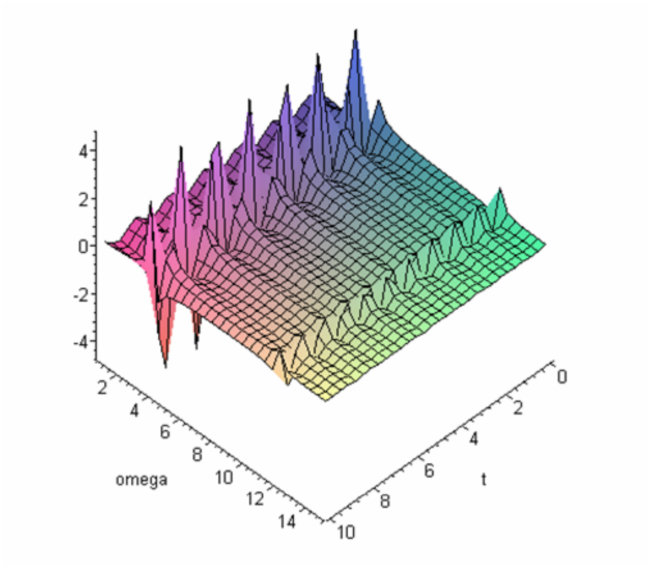


fig. 9

The fig. 9 only shows the first two resonance peaks. To be accurate these peaks undouble. As we can better see from the fig. 10 and 11, we have these peaks in correspondence of

$$\omega = \boxed{1} \times 4 = 4 \quad e \quad \omega = \boxed{3} \times 4 = 12. \tag{4.7}$$

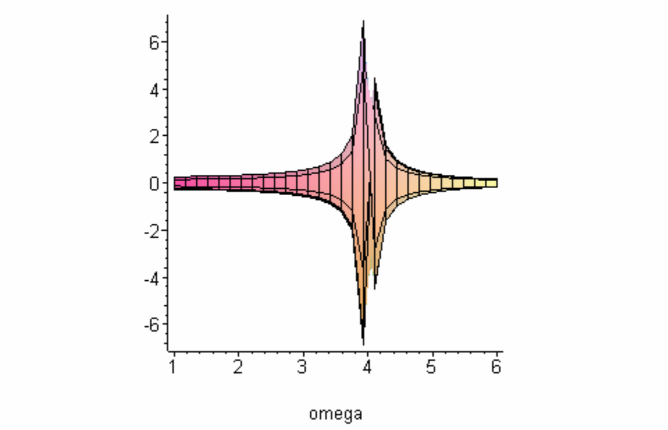


fig. 10

In correspondence of $\omega = 4$ we have a maximum oscillation ampleness equal to ± 6.4 units. From the fig. 11 we have, in correspondence of $\omega = 4 \times 3 = 12$, that

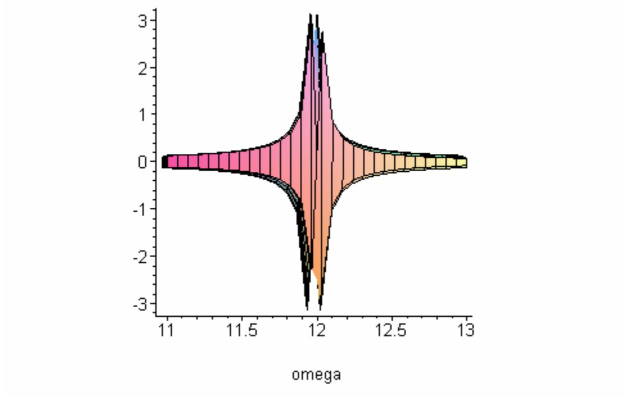


fig. 11

the said ampleness is equal to ± 3 units.

Now let's calculate the answer of the same oscillator by hypothesizing rigorously sinusoidal forcer.
In that case the stationary part is given by the formula (3.4).
The figures 12, 13 and 14, that follow, let us to evaluate the answer of the dumping harmonic oscillator subjected to the said forcer. We assume the same values of the previous case and so

$$\Omega = 4, \quad \delta = 10, \quad g = \gamma = 0.01 . \tag{4.8}$$

In that case it's evident that application time of the force is equal to the entire period.

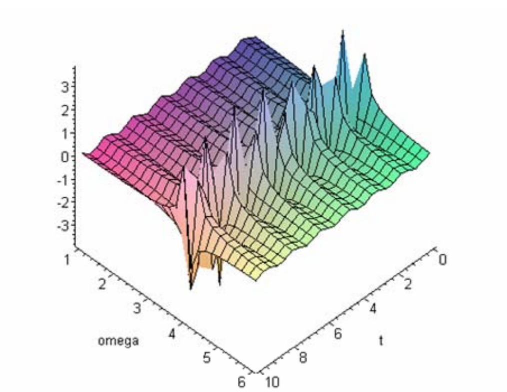


fig. 12

From the fig. 12 we evict, as it's known, the unique resonance condition[6] $\Omega \cong \omega$. The fig. 13 let us to evaluate the mass movement, that is equal to ± 3.6 units, much more little of the one had in the previous case.

[6] Effectively, the presence of a not so much strong attriction slightly moves the resonance conditions: In that case the unique condition comes when ω does not coincide exactly with Ω $(\omega \approx \Omega)$.

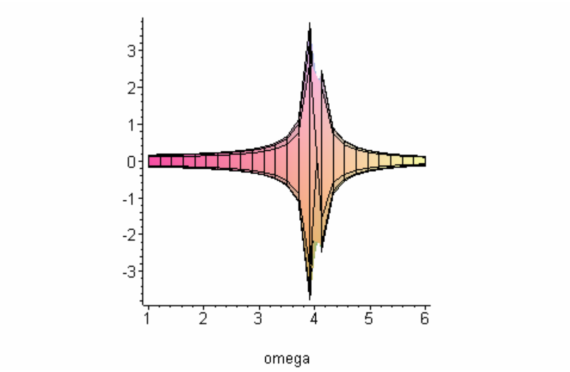


fig. 13

The fig. 14 let us the evaluate the movement we have in correspondence of $\omega = 12$.

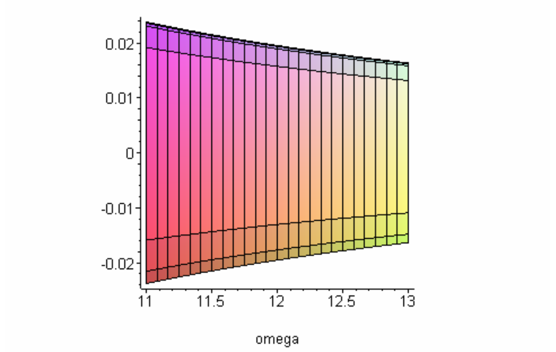


fig. 14

It's equal to 0.02 units, against the previous value equal to 3 and so about 150 time littlest. It's opportune, even for what we're going to see, to consider the example of a forcer of this kind

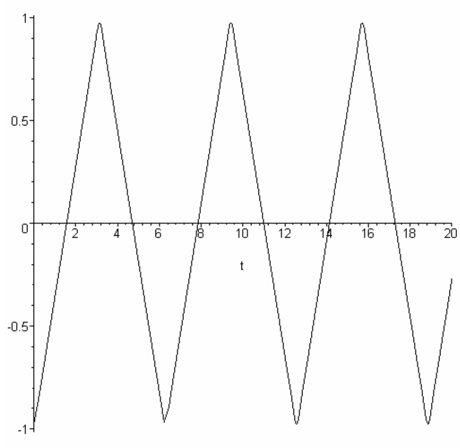


fig. 15

so the corresponding equation to solve is

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega^2 x = -\frac{8}{\pi^2}\delta\left[\frac{1}{1^2}\cos(1\Omega t) + \frac{1}{3^2}\cos(3\Omega t) + \frac{1}{5^2}\cos(5\Omega t) + .. + \right] \qquad (4.9)$$

It, this time posed

$$\mathrm{T}_n = -\frac{8\delta}{\pi^2 n^2}, \qquad (4.91)$$

has the analog stationary solution

$$x(t) = \mathrm{T}_n \frac{\cos(n\Omega t - \Theta_n)}{\sqrt{\left(n^2\Omega^2 - \omega^2\right)^2 + n^2\gamma^2\Omega^2}} \qquad \left(n = 1, 3, 5, 7, \ldots\right).$$

Fig. 16, always for

$$\Omega = 4, \quad \delta = 10, \quad g = \gamma = 0.01,$$

poses in evidence the resonances for $\omega = 4 \quad e \quad \omega = 12.$

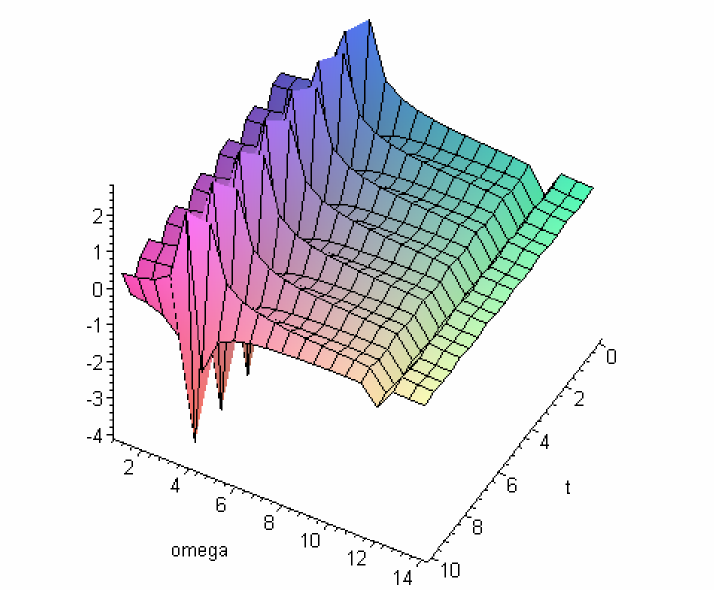


fig. 16

Fig. 17 and 18 let us to evaluate the entity of the oscillation amplenesses.

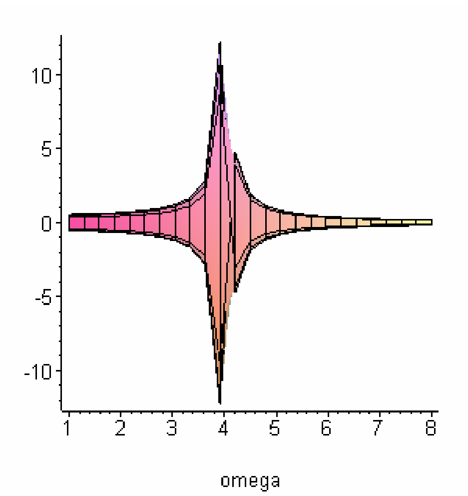


fig. 17

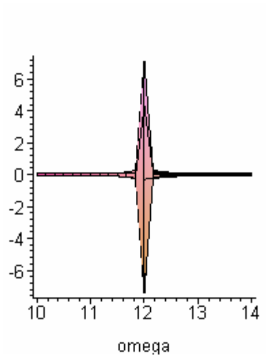


fig. 18

Finally, as we've just seen, the answer of the oscillator subjected to this kind of forcers, can be posed in the form (harmonic analysis)

$$x(t) = \frac{\mathrm{T}_n}{\sqrt{\left(n^2\Omega^2 - \omega^2\right)^2 + n^2\gamma^2\Omega^2}}$$

represented in the fig. 20.

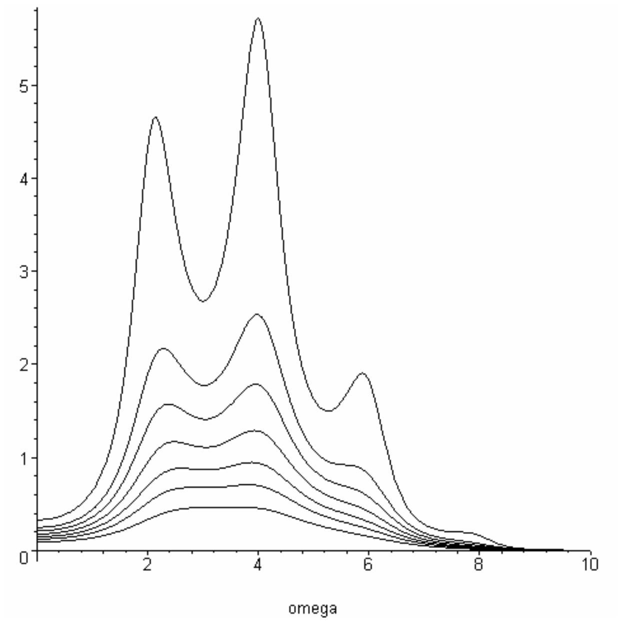


fig. 20

Resonance verifies everytime $\omega \approx \Omega n$. The highest and external curve to the others is caused by a very little value of the oscillator attrition. Little by little γ increases, resonance peaks tend to alloy until their nearly desappearment . the fig. 21 only shows the shortest curve (dissipation of the highest energy) of the fig. 20.

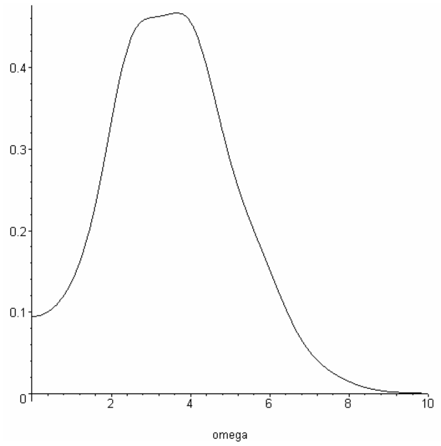


fig. 21

Instead if we consider the classic equation

$$\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+\omega^2 x=\delta\cos(\Omega t)\,, \tag{4.10}$$

which stationary solution (harmonic analysis) is

$$x(t)=\frac{\delta}{\sqrt{\left(\Omega^2-\omega^2\right)+\Omega^2\gamma^2}}\,, \tag{4.11}$$

we have the graph of fig. 22.

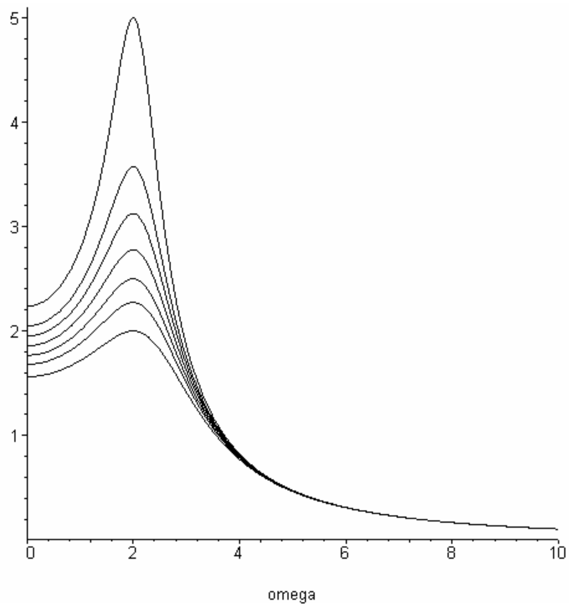


fig. 22

5. Some experimental considerations

To experimentally verify resonance phenomenon, is commonly used the dispositive shown in fig. 23. It consists in a beam hinged to the tailpieces in which middlepart is posed a badminton with an oddball mass so that badminton is not balanced.
By the use of variable angular velocities, we experimentally verify that when the pulsation Ω of the oddball mass tends to coincide with ω of the beam itself, the amplenesses (outlined in the figure) increase more and more until the break of the beam itself. In that case it's applied a rigorously sinusoidal forcer to the oscillator.

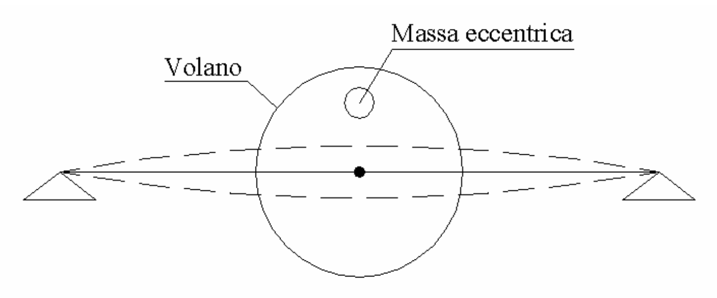


fig. 23

The fig. 24, taken by the work [1], shows the case the said badminton is applied to the top of

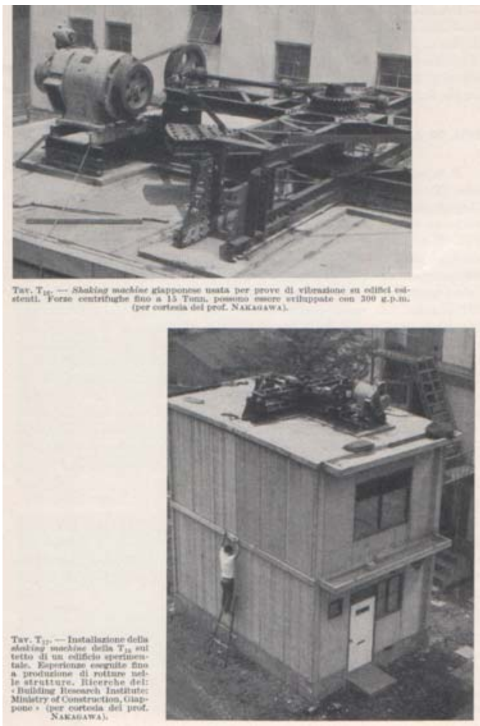


fig. 24

a structure to be studied.

It's the case to observe that an operator applying a forcer as those previously examined anyway produces multiresonance effects.

For example let's consider the (project of) bridge on the strait of Messina. It's previsedn a central bay of ml. 3.300.

If we pose in the middle of this bay a badminton with a vertical rotation axis, the said bay will surely come horizontally in resonance only when $\omega \approx \Omega$[7], infact the forcer is still rigorously sinusoidal. But if the said structure is invested by blasts of wind with pulsation Ω, surely the dangerous resonance phenomenon will occur when the it's true the identity

$$\omega \approx \Omega n \qquad (n = 1, 2, 3..).$$

Remains the question if seismic waves can legitimately approximate with sinusoidal forcers or if they are an impulsive phenomenon. About this [1, page. 305], we textually read:

The results of the researches led but the Institute of Technologies of Pasadena have been object of a lot of discussions. American Seismologists retain that only through the suggestive hypothesis that the earthquake, afar from being a <continue> physic phenomenon, would be constituted by a series of impulses, we can have count of its fundamental irregularity, infallibly noticeable by spectral cinematic.

So it's evident that if the accelerogrammas registered during telluric tosses are read with *continuity* so there is resonance, for a system with a single degree of freedom, only when $\Omega = \omega$. Instead if those ones are assimilated at impulses, in that case we have the just said multiresonance.

[7] In the hypothesis of linear comportment of the structure.

6. A possible reading of the seismogram.

The forcer to be applied to a damping oscillator is deducted by a generic seismogram.

For simplicity, let's take in consideration an example taken by the text [3].

The fig. 25 shows the punctual accelerations, in function of time, to which is subjected a damping harmonic oscillator.

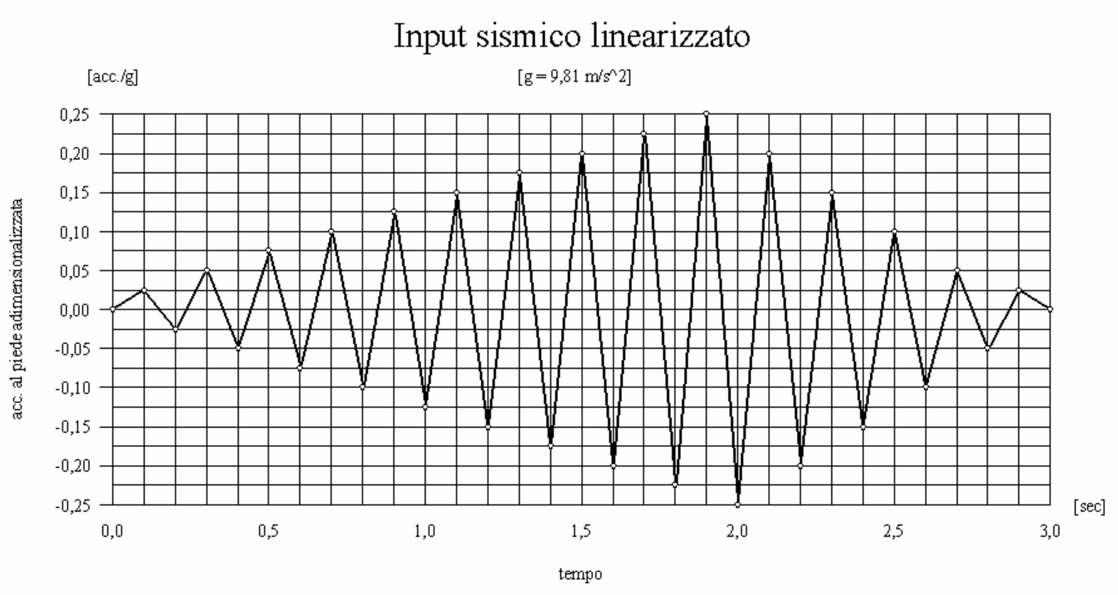


fig. 25

It's known the answer is evaluated by a numeric integration[8], through Duhamel's integral [2,3,4]. In basis to what previously said and in particular in the case of the forcer of the fig. 25, it's immediate to see, because the pulsation of that forcer is constant and equal to $\Omega = 2\pi / 0.2 = 31.41$, that, by a Fourier approximation, even those oscillators with a pulsation equal to a multiple entire uneven of it go in resonance.

Certainly we don't know how seismic waves are intimately composed so any restrictive hypothesis about their nature can provoke big damages, but on the other hand we can certainly affirm that a real oscillator can go in resonance only everytime is verified the most general condition

$$\omega \simeq \Omega\, n \quad (n = 1, 2, 3, ..) \tag{1.1}$$

so if we neglect this thing we are exposed to unpredictable and enormous risks.

There's, by the writer's side, a clamorous precedent physic that corroborates what is said in this issue and regards the note Problem of the Black Body, for all simile to the seismic one.

[8] Which limits, very strong, are well known.

It consists in establishing with what a physic appliance the luminous energy[9] (phenomenon eminently oscillating) is absorbed by matter (formed by an infinity of harmonic oscillators to which are assimilated the atoms forming the wall of the Black Body[10]). To arrive to a theoretical justification of a known formula and experimentally deducted[11], the famous dutch physic M. Planck was led and constricted to formulate, pain the missing justification of the said experimental relation, the incredible[12] hypothesis that energy would be formed by invisible packets. For exactness He was constricted to expressly admit that a *harmonic oscillator* (atom) cannot absorb energy with continuity, but per packets or invisible capsules. This *postulate*[13] is expressed by His *still empiric* and as much famous relation[14] [15]

---

[9] For exactness the energy of the electromagnetic waves.

[10] Here things are more simple. Infact the various harmonic oscillators costituting the wall of the black Body (atoms) can be considered disconnected among them. Instead, in the case of a structure, we have distribuited masses and elastically connected among them.

[11] We should mistrust of equations deducted in that way. Infact to the experiment can always escape something; more accurate instruments can pose in evidence unimaginable effects. The formula that describes with a good approximation the experiences produced in laboratory (justified later by Planck) is the known relation

$$E_\lambda = hC^2\lambda^{-5}\frac{1}{\exp(\frac{hC}{kT\lambda})-1}.$$

The formula (obviously empiric) that better represents the experiments made is instead that one by Pringsheim-Lummer (see H. Kangro, **Early History of Planck's Radiation law**, Taylor & Francis L.t.d.,London (1976)

$$E_\lambda = hC^2\lambda^{-5}\frac{1}{\exp\left(\frac{hC}{kT\lambda}\right)+\exp\left(-\frac{kT\lambda}{hC}\right)-1}.$$

It's anyway to be observed the real curve (see P. Rossi **Storia della Scienza** Utet Vol. III (*) pag. 94) presents some showy depressions simile to the *delves* of fig. 20, deformations that are not reproduced by these relations and that are, likelyway, they are just caused by multiresonance.

[12] The first one who did not believe it was Planck himself, it was at the beginning of the past century. As it's known He spent the whole rest of his life to find a reasonable explqanation of this abstruse hypotesis. Until our days nobody has succeeded in finding it. Later, through the years, this incredible thing came to be passively accepted, and this is because both for the human and the merciful spirit of the accustoming and to the next interpretation, if we can call it this way, of other experimenal facts.

[13] The postulates of a Theory pretending to interpret the natural phenomenas should be immediately clear and evident and this for postulates' definition itself: this is the fundamental rule of a Theory. Instead, very often, the blastoff affirmations, even if correct, are real theorems to show. And it's not only a pure question of gnoseologic comprension (E. Mach invoked, in addiction, thought economy) because we run the big risk to see the reality through a very distorted lens,and that's worse, an incognito and congenital degree of myopia.

[14] From what the actual Quantum Mechanic was born.

[15] In the case of the elastic bodies we hypothesize the existence of the phonons, in analogy with electromagnetic evnergy photons (6.1). On purpose we observe that elastic force is amenable to the electric force that alloys the atoms of a substance. Infact, is possible to show [6] that the elasticity module of any homogeinic substance is deducible by the perfectible relation

$$E_{Young} = \frac{2e^2}{d^4}\,[dyne/cm^2],$$

where e is the electron charge and d is the interatomic distance, this last one determined by the spectroscopic analysis of the substance. Analog relations also subsist that allow to determ the coefficient of thermal dilatation and the sound's speed in the considered matter, all amenable to the electron charge, to the mass and to the said distance. In other terms, among macroscopic (that are actually determed only in laboratory) and microscopic bignesses exist some evident bonds.

$$E = h\nu n \qquad (n = 1, 2, 3...)\,, \tag{6.1}$$

where h is Planck's constant, ν is the frequency of the electromagnetic radiation and n is a rigorously entire number, founding relation of the actual Quantum Mechanics.

Now this enigmatic formula[16] (6.1) can also be written, by the introduction of pulsation, so

$$E = h\nu n = h\frac{\Omega}{2\pi}n$$

and, posed, as usual

$$\frac{h}{2\pi} = \hbar\,,$$

even

$$E = \hbar\Omega n.$$

But from this equation we can elicit the relation $E/\hbar$, that evidently represents another pulsation, so we have

$$\frac{E}{\hbar} = \omega = \Omega n\,,$$

formula that we already know.

So, (attended that the energy crossover between two vibrant systems can only be regulated by resonance phenomenon), we can affirm that presumed discontinuity of the energy – (caused by the presence of the as much enigmatic entire numbers) – would be amenable to a simple, immediate and comprehensible multiresonance resonance ?

[16] By it would be arguable, in addiction, that the daily macroscopic reality would mask a real microscopic discontinuity for the littleness of the Planck's constant.

7. Seismic energy assimilation.

A cluster of seismic solicitations (or as other analog), surely of random[17] nature, can be constituted by any type of waves with multiple armonicas.
A complex structural system can be assimilated, at the first instance, by a complement of harmonic oscillators, disconnected among them. It's anyway possible to impose among them the condition of spacial congruence.

Because a damping harmonic oscillator can go in resonance, as seen, only whenever is verified the condition

$$\omega \cong \Omega n \qquad (n = 1, 2, 3...),$$

it's opportune to evaluate the assimilation modalities of seismic energy by its side, in the simplest hypothesis it would be unique. If, for example, we consider the (3.2), we can calculate the absorbed energy with the relation

$$E = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{dx}{dt} \right)^2 . \tag{7.1}$$

For it we have the following condition of stationariness

$$E = \frac{1}{2} m \left[\sum_{n=1}^{n=\infty} \frac{(-1)^n \frac{2\delta}{\pi n} \sin(na)\, n\Omega \sin(n\Omega t - \Theta_n)}{\sqrt{\left(n^2\Omega^2 - \omega^2\right)^2 + n^2\gamma^2\Omega^2}} \right]^2 . \tag{7.2}$$

In concomitance of the resonance we obtain

$$n^2\Omega^2 = \omega^2 \quad \Rightarrow \quad \Theta_n = \arctan\left(\frac{n\Omega\gamma}{n^2\Omega^2 - \omega^2} \right) = \frac{\pi}{2} \tag{7.3}$$

so, by neglecting multiple products that generate other assimilation lines, we simply have

$$E_{\max} = \frac{1}{2} m \left(\frac{2\delta}{\pi\gamma} \right)^2 \sum_{n=1}^{n=\infty} \frac{\sin^2(na)\sin^2(n\Omega t - \pi/2)}{n^2} = \frac{1}{2} m V^2 \sum_{n=1}^{n=\infty} \frac{\sin^2(na)\cos^2(n\Omega t)}{n^2} . \tag{7.4}$$

Took count that

$$\Delta t = \frac{2a}{\Omega} \quad e \quad \omega = \Omega n \tag{7.41}$$

the (7.4), posed

[17] Effectively, if it's true we don't have the possibility to certainly formulate some hypothesis about the composition of any periodic solicitation, it's otherwise true that an haromic oscillator can go in resonance only under the condition $\omega \cong \Omega n$. But we can say the the damping haromic oscillator act as a filter, by letting itself to be crossed by certain solicitations and by capturing some others.

$$V = \frac{2\delta}{\pi\gamma} \tag{7.5}$$

becomes

$$E_{\max} = \frac{1}{2} m V^2 \sum_{n=1}^{n=\infty} \frac{\sin^2\left(\frac{\omega}{2}\Delta t\right)\cos^2(\omega t)}{n^2} \tag{7.6}$$

so the maximum generic term of the energy is given by the expression

$$E_n = \frac{1}{2} m \bar{V}^2 \frac{1}{n^2} \tag{7.61}$$

having posed

$$\bar{V} = V \sin\left(\frac{\omega}{2}\Delta t\right). \tag{7.7}$$

The fig. 26, that follows[18], denounces under which pulsations the oscillator with its pulsation ω, absorbs energy (formula 7.2).

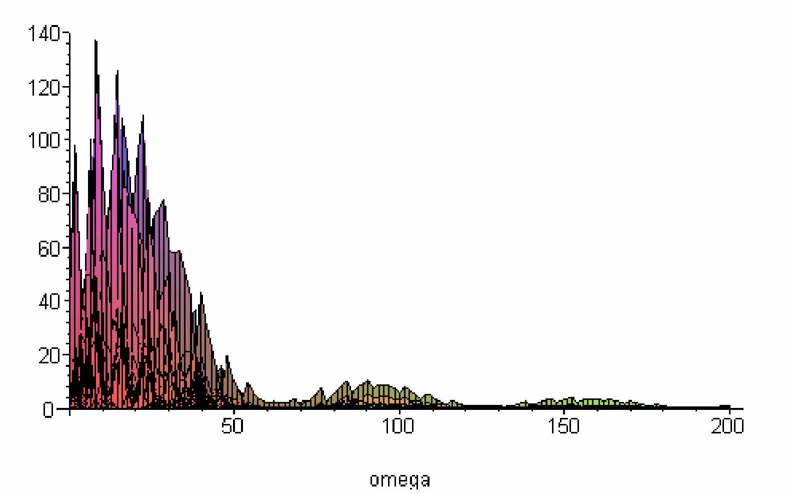


fig. 26

It's opportune to observe that the various assimilation lines thicken around particular pulsations (fine structure).

Let's consider the harmonic ones of (7.2) we have

$$E = \frac{1}{2} m \left[\sum_{n=1}^{n=\infty} \frac{(-1)^n \frac{2\delta}{\pi} \Omega \sin(na)}{\sqrt{\left(n^2\Omega^2 - \omega^2\right)^2 + n^2\gamma^2\Omega^2}} \right]^2. \tag{7.8}$$

[18] In this figure we put Ω and we made ω to vary.

The figures that follows[19], for various crescent values of γ, allow to have an idea of (7.8).

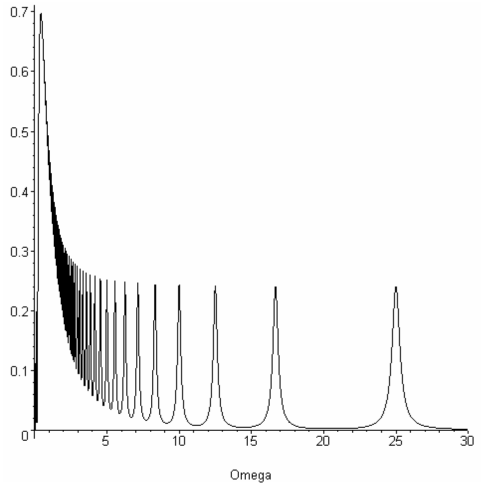


fig. 27

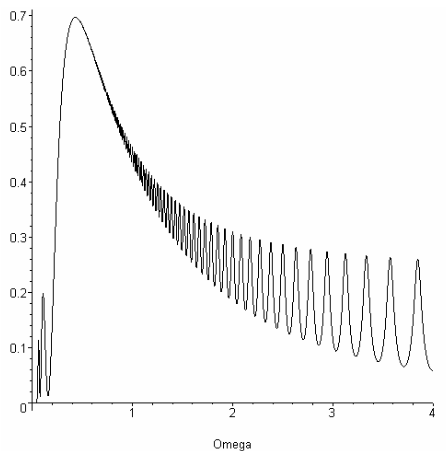


fig. 28

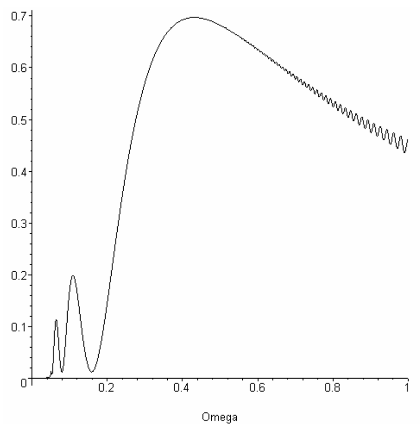


fig. 29

[19] Instead in the following figures we put the value ω and we made Ω to vary.

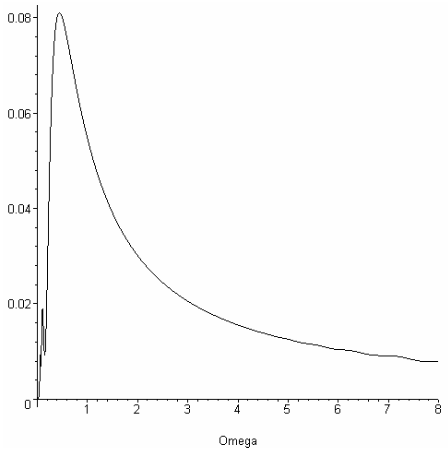


fig. 30

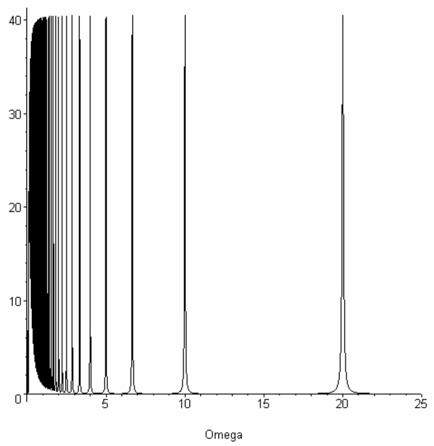


fig. 31

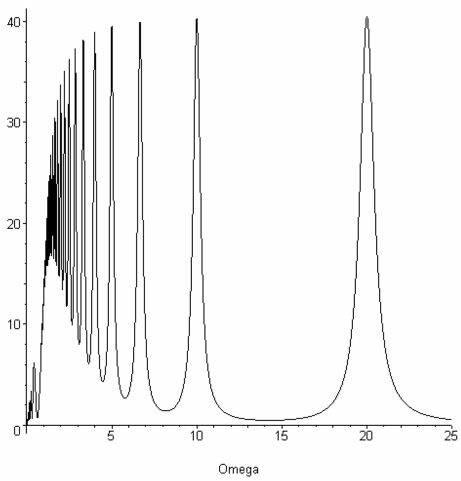


fig. 32

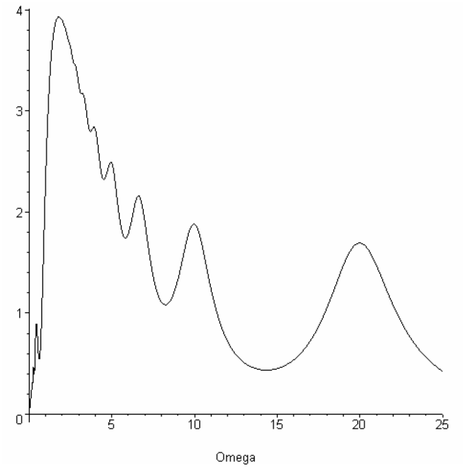


fig. 33

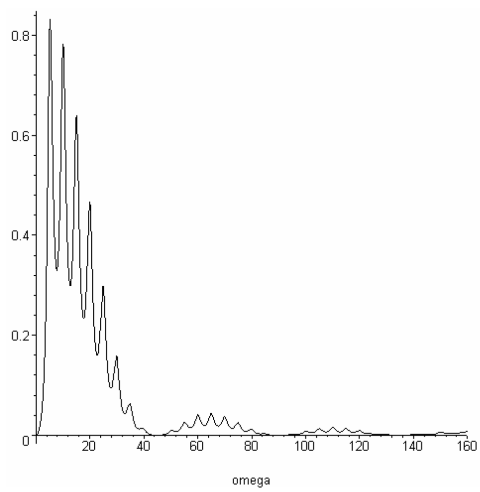
The following pages represent instead the formula

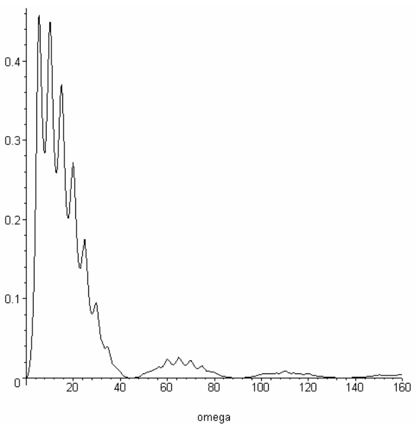
$$E = \frac{1}{2} m \sum_{n=1}^{n=\infty} \left[\frac{(-1)^n \frac{2\delta}{\pi} \Omega \sin(na)}{\sqrt{\left(n^2\Omega^2 - \omega^2\right)^2 + n^2\gamma^2\Omega^2}} \right]^2 \tag{7.9}$$

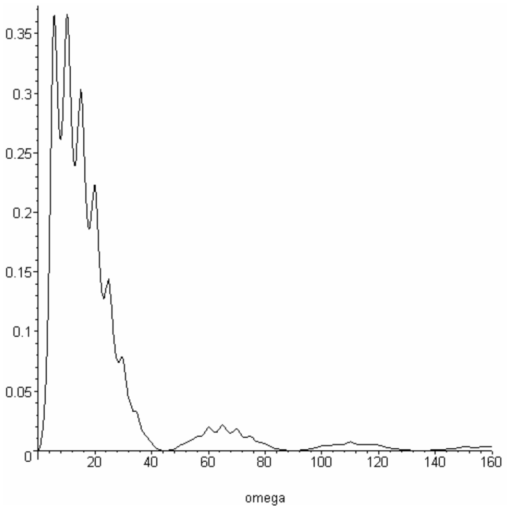
that neglects multiple products, puts the value Ω having ω as variable.

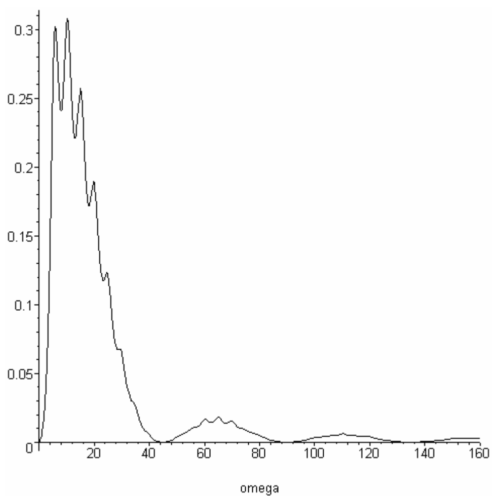
The graphs below represent the (7.9) for the following values

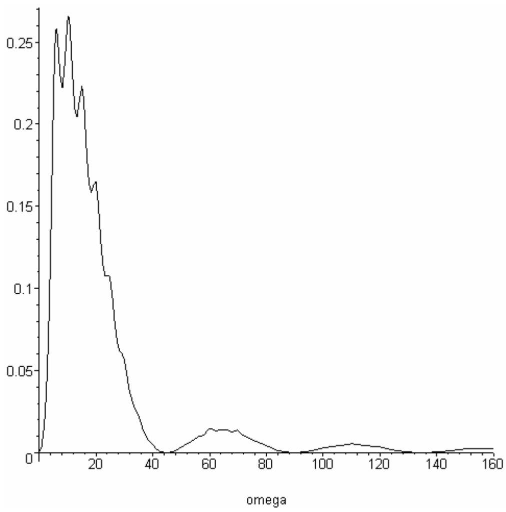
$$\delta = 10 \quad a = 0.07 \quad \Omega = 5 \quad \gamma \in [2.5,\ 10.5] \tag{7.10}$$

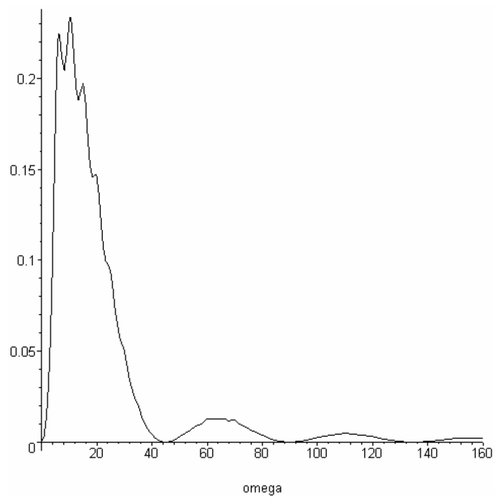


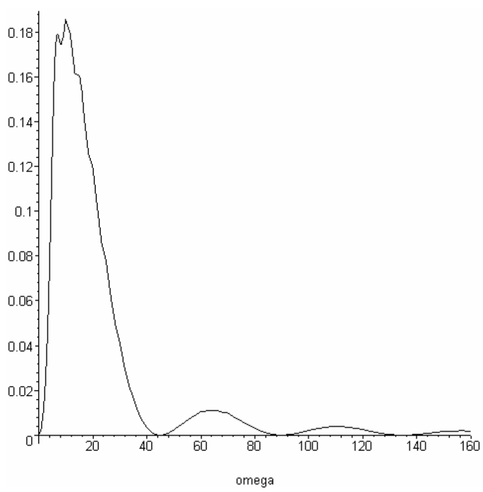


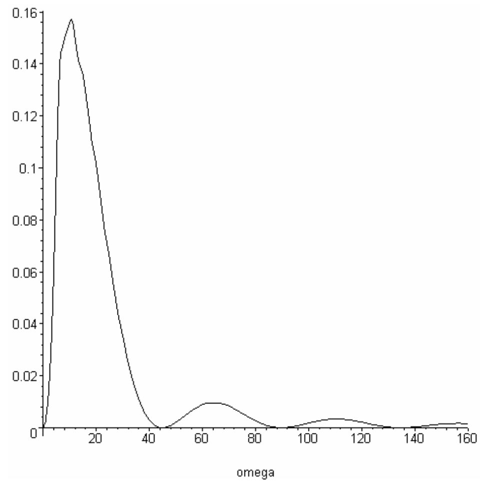

omega

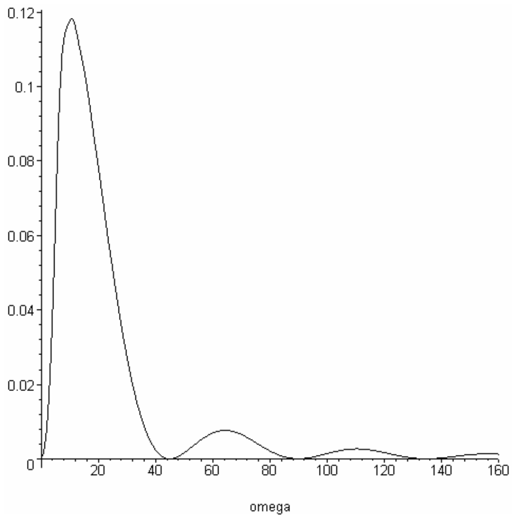

omega

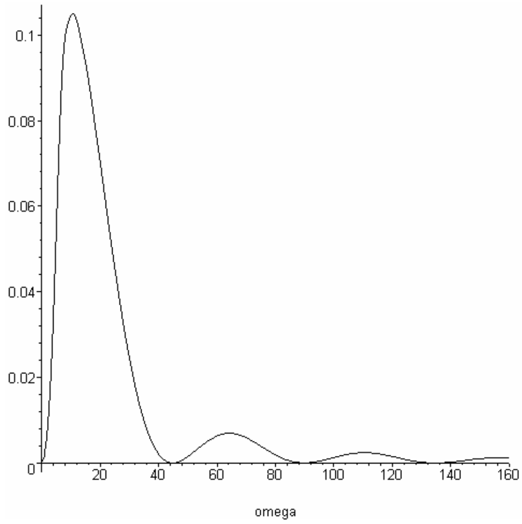

omega


omega


omega


omega





Even here we note some assimilation lines transforming in sinusoids.
We can observe how the various assimilation lines, well evident in the first graphs, little by little the value of γ increases, they tend to disappear. Viceversa, for the values of γ smaller and smaller, the various lines tend to assume a unique energetic value.
Fig. 34, below

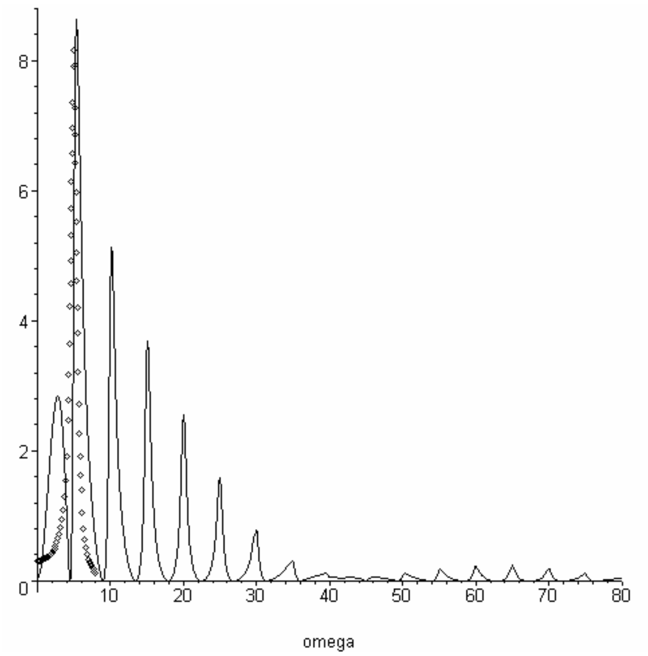


fig. 34

shows, by a continuate tract, the (7.9) and, represented per dots, the classic formula

$$E = \frac{1}{2} m \left[\frac{\delta \, \Omega}{\sqrt{\left(\Omega^2 - \omega^2\right)^2 + \gamma^2 \Omega^2}} \right]^2 \qquad (7.11)$$

As we can see from the confront, while with the (7.9) we have a big assimilation regarding all the numerable pulsations of the break $\omega \in [0, \infty]$, with the (7.11) we have a unique line. From what we perceive another new solution for the Problem of the Black Body.

So if a single harmonic oscillator has infinite and numerable resonance conditions to which correspond different grades of energy assimilation, we can say that it's valid no more The Energy Equipartition Theorem[20] drawable from Classic Mechanics, or better, furthermore precised. Let's take in consideration the fig. 32. In it there's an oscillator with its own pulsation ω, subjected to an external forcer with pulsation Ω and this one variating with absolute continuity. From the said figure we evict the oscillator instead absorbs energy in a discrete way. When Ω is null the absorbed energy is either null. Little by little that Ω increases the extreme peaks of the absorptions tend to increase until they arrive to a constant asymptotic value. So, for crescent values of Ω and for the elected values of γ, in this case the said absorption line peaks tend to balance. So we can say that for elevated values of Ω and only for a particular value of γ we find again as a limit case the said Principia.

In addiction, if we admit that the electromagnetic radiation is representable by a forcer of kind (3.1) and we pose, in the case of hydrogen atom,

$$\bar{V} = \frac{C}{137} \tag{7.12}$$

then the (7.6) coincides with Bohr's relation. In that case the (7.6) becomes

$$E_{\max} = \frac{1}{2} m \frac{C^2}{137^2} \frac{1}{n^2} \tag{7.13}$$

where C is the light's speed, m is the electron mass $1/137$ is the fine constant structure. Because it's known that

$$2\pi 137 e^2 = hC, \tag{7.14}$$

where e is the electron charge and h is Planck's constant, the (7.13) becomes

$$E_{\max} = \frac{2\pi^2 m e^4}{h^2} \frac{1}{n^2} \tag{7.15}$$

that is the famous Bohr's relation. But He obtains the (7.15) with a filled series of postulates in open contradiction with heavy experimental facts, foreseen from the consolidated classic mechanics, and by the use of a mathematical expedient not yet fell in disuse at all. Later Bohr confided to the young Heisenberg [8] those facts and His deep skeptisism in the impossibility of the Classic Mechanics to interpret microscopic phenomenas. In a concise way, we show the *new and strong postulates* that He had to assume to *explain* hydrogen atom lines.

[20] By this Principia, that generates the famous Ultraviolet Catastrophe (Rayleigh & Jeans), solved by Planck by the hypothesis of the energy quantum, energy would equipart among all the oscillators. It is elicited in Classic Mechanics by considering the known harmonic oscillator, deprived of resistence and with a unique condition of resonance.

- With Bohr occurs to admit that electron can rotate around the nucleus only on particular orbits in biunique correspondence with complement of the entire numbers[21].
- It occurs to admit that the electron, even by running on its particular circular orbits of ray r_n, does not emit radiations and this in open an unsolved contrast with the cleared fact that an accelerated charge emits electromagnetic waves.
- It occurs, most precisely, to admit that the action endured by the said charge from the nucleus would not have any value (as it happens in Classic Mechanics) but it would be an exact multiple of the action h, that appears in the empiric formula of Planck (6.1).
- It occurs to admit that exist a postulate, called *mechanical,* by $E = h\nu n$ and another postulate, called optical or maimed that is still in use, by it instead $E = h\nu$. Instead if we coherently we always and solely admit the (6.1), as it's easy to verify, we obtain a different relation from the (7.12), that is in contrast with the experimental facts: at the place of n^2 we have n^3. This is the banal mathematical expedient.

When Bohr submitted His theory to the Authoritative Ernest Rutherford he had this answer:

Dear Dr. Bohr

I received your paper and I read it with great interest.. . Your ideas about the origin of the hydrogen spectrum are very clever and seem to work fine, but the mixing of Planck's ideas with the old mechanics makes it difficult to obtain a physical idea on which the whole argument should be based. …

20 March 1913

E. Rutherford

If we consider the generic term

$$x(t) = \Psi_n = \frac{T_n}{\sqrt{\left(n^2\Omega^2 - \omega^2\right)^2 + n^2\gamma^2\Omega^2}} \qquad (7.16)$$

in resonance conditions $(n\Omega = \omega)$ we have, for the (7.7),

$$\Psi_n = \frac{T_n}{\pm\gamma\left(n\Omega\right)} = \frac{(-1)^n \frac{1}{n}\frac{2\delta}{\pi}\sin(n\,a)}{\pm\gamma\,\omega} = \frac{\left(\frac{1}{\gamma}\frac{2\delta}{\pi}\sin(\frac{1}{2}\omega\,\Delta t)\right)}{n\,\omega} = \frac{\left(\frac{C}{137}\right)}{2\pi\,\nu\,n} = \frac{1}{2\pi 137\,n}\frac{C}{\nu} = \frac{\lambda}{2\pi 137 n}, \qquad (7.17)$$

where C is the light's speed and λ is the electromagnetic wave length. From what we have the new relation that alloys the resonance length of the charge to the electromagnetic wave length it generates[22] that is [9]

[21] Instead in Classic Mechanics these rays can assume with continuity all the values in the field of the real numbers.

$$\lambda = 2\pi 137\,\Psi_n\, n\,. \tag{7.18}$$

The same relation can be obtained independently from what said in the present work [9]. From (7.18) follows that Ψ_n coincides with the Bohr's ray r_n. Infact by multiplying the (7.18) for the frequency ν we have

$$\lambda\nu = C = 2\pi 137\,\Psi_n \nu\; n = 137\,\Psi_n\,\omega\; n = 137\, v_n\, n \tag{7.19}$$

from what follows

$$v_n = \frac{C}{137\, n} \tag{7.20}$$

On the other hand, in the case of the dipole proton-electron, we can write that

$$v_n = \sqrt{\frac{e^2}{m\Psi_n}} \tag{7.21}$$

and by equalising these last equations we have

$$\Psi_n = \frac{e^2}{mC^2} 137^2 n^2 = R_e\, 137^2 n^2 \tag{7.22}$$

by having denoted with R_e the classic ray of the electron. Took count of the identity (7.14), we have that the (7.22) can be written

$$\Psi_n = \frac{h^2}{4\pi^2 e^2 m} n^2 \tag{7.23}$$

that coincides with Bohr's relation. Had on mind that the (7.18), by introducing de Broglie's relation, can be written

$$\lambda = \lambda_{dB}\, 137\; n \tag{7.24}$$

we have a bond between the electromagnetic wave and the de Broglie's one.

By this we also solve the weighty and omnipresent paradox wave-corpuscle. Infact the electron, or any charged particle, has intrinsically the familiar corpuscular aspect, when it's free. In the case it would be part of a dipole, it is constricted, from the resonance phenomenon, to vibrate in perfect syntony with the electromagnetic wave and therefore it acquires, in some circumstances, all the features of the undulatory phenomenas. Finally it's to be observed if we assume 137 equal to the unit de Broglie's wave identifies with the electromagnetic one.

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[22] Both in classic electrodynamics and in quantum one we admit that the electromagnetic radiation frequency coincides with the one of the dipole generating it. Nothing is said about the eventual bond between Ψ of the charge vibration and and the electromagnetic wave length λ it generates.

The relation (7.24) can be verified remaking the experience verifying de Broglie's relation and also by measuring the electromagnetic radiation that evinces during the same experiment.
So we can say that de Broglie's incredible mute orbits are this way because they just constitute the moments the electromagnetic wave is absorbed by matter.

So it's possible to specialize the solution of the (3.1) in the case of the hydrogen atom, by determining the value to attribute to the bignesses γ, ω, δ e Δt. With it is finally possible to describe with the maximum precision all the mechanical features that intervene in the interpretation phase between the wave and the atom (hidden variables). So it's interpreted the relation of indetermination by Heisenberg because now it's more comprehensible the existence of those hidden variables hypothesized by Einstein, arduous adversary of the physic laws casualness (God does not play dice).

If, for example, the position (7.12)

$$\frac{2}{\pi}\frac{\delta}{\gamma}\sin(na) = \frac{2}{\pi}\frac{\delta}{\gamma}\sin(\frac{1}{2}na) = \frac{2}{\pi}\frac{\delta}{\gamma}\sin(\frac{1}{2}\Omega\, n\Delta t) = \frac{2}{\pi}\frac{\delta}{\gamma}\sin(\frac{1}{2}\omega\Delta t) = \frac{C}{137}$$

Is furthermore precised by posing

$$\frac{\delta}{\gamma} = C \tag{7.25}$$

and

$$\frac{2}{\pi}\sin(\frac{1}{2}\omega\Delta t) = \frac{1}{137}, \tag{7.26}$$

we have what follows. From the (7.25) we deduce that

$$\frac{\delta}{\gamma} = \frac{\bar{V}\omega}{\gamma} = \frac{C}{137}\frac{\omega}{\gamma} = C \longrightarrow \gamma = \frac{\omega}{137}, \tag{7.27}$$

so the damping is less than the pulsation. From the (7.26) we have that the duration of the electromagnetic impulse is given by

$$\Delta t = \frac{1}{2}\frac{T}{137} \tag{7.28}$$

where T indicates the oscillator period. Therefore the external impulse duration is 274 more little of the time needs the electron to run its orbit.

It's to observe that by actual knowledge [11], differently from what we deduce by the (7.18), we only have that the electromagnetic wave length is a lot greater than electron ray [11, p. 27] that is

$$\lambda = \frac{C}{\nu_o} \gg \frac{e^2}{mC^2} \tag{7.29}$$

and that, differently from the (7.5),

$$\gamma \ll \omega \,. \qquad (7.30)$$

These are the first results obtained ***if we want*** to interpret the empiric relation of Planck as a more coherent resonance condition, although more general of the one always silently hypothesized in theorical physics .

Until here we have given some becks about the electromagnetic dipole. In an analog way can also be considered the gravitational dipole (binary system M + m) [7], with an evident and immediate generalization even of the (7.18). Infact this one can be written in a more general form

$$\lambda = 2\pi \frac{C}{V} \Psi n \qquad (7.31)$$

by indicating with V the charge medium velocity or the mass in the assimilation phase. If the gravitational wave that bangs the dipole has a speed equal to the one of the light, then the (7.10 or 7.31) specializes. In that case we have that the fundamental wave length $\bar{\Psi} = 2\pi\Psi = 2\pi GM / C^2$ is equal to the semi-length of Schwarzschild's circumference[23]. This could be the right way to solve the actual problem of the gravitational energy quantization.

But maybe it could be opportune first to reconduct gravity to already known interactions !

For completion, it occurs to relieve that even if we consider the non linearity of the electric field [5] (non linear or keplerian oscillator[24]) that is the equation

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \frac{\omega^2 x}{\left(1 - \varepsilon \frac{x}{p}\right)^2} = \delta \exp\left(-i\Omega t\right) \qquad (7.32)$$

we find again the multiresonances of the single oscillator. Infact the stationary solution of the (7.32) is of the type [7]

$$x(t) = \sum_{n=1}^{n=\infty} \frac{\mathrm{T}_n \omega^2}{\left(n^2\Omega^2 - \omega^2 + i\gamma n\Omega\right)} \exp(-i\,n\,\Omega\,t)\,. \qquad (7.33)$$

[23] It's to note as a rule of the classic ray of the electron of the (7.14) is assumed from the one of Schwarzschild.
[24] With whom we have another solution of the problem of the Black Body.

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